RAJALAKSHMI ENGINEERING COLLEGE (AUTONOMOUS)
THANDALAM, CHENNAI – 602105.

PH19142

PHYSICS FOR BIOSCIENCE

Common to I semester- B.E. – BME and B.Tech - Bio.Tech & Food Tech

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Faculty of Physics
Department of Humanities and Sciences

PH19142 - PHYSICS FOR BIOSCIENCE

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Common to I sem. B.E. – BME and B.Tech - Bio.Tech & Food Tech

OBJECTIVES

- To enhance the fundamental knowledge of Physical, Quantum, magnetic and dielectric properties of materials
- To study the behavior of light, sound and nuclear radiation in materials.

UNIT I - PROPERTIES OF MATTER 9

Elasticity – Stress-strain diagram and its uses - factors affecting elastic modulus and tensile strength – torsional stress and deformations – twisting couple - torsion pendulum: theory and experiment - bending of beams –area moment of inertia - bending moment – cantilever - applications – uniform and non-uniform bending- I-shaped girders - stress due to bending in beams.

UNIT II - QUANTUM PHYSICS AND SUPERCONDUCTIVITY 9

Introduction to wave function - derivation of Schrodinger wave equation -Particle in a three dimensional box – degenerate states – Fermi- Dirac statistics – Density of energy states – Electron in periodic potential – Energy bands in solids - Tunneling -scanning tunneling microscope. - Introduction of Superconductivity - Properties of Superconductors - Meissner Effect - BCS theory (qualitative) - Type-I and Type II Superconductors - Magnetic Levitation and SQUID.

UNIT III - MAGNETIC AND DIELECTRIC MATERIALS 9

Magnetism in materials – magnetic field and induction – magnetization - magnetic permeability and susceptibility – types of magnetic materials – microscopic classification of magnetic materials – Ferromagnetism origin and exchange interaction - saturation magnetization and Curie temperature – Domain Theory- M versus H behavior – Hard and soft magnetic materials - Introduction electrostatics and EM waves – Tissue as a leaky dielectric - Relaxation processes: Debye model, Cole–Cole model.

UNIT IV – WAVES, OPTICS, AND SOUND 9

Oscillatory motion – forced and damped oscillations: differential equation and its solution – plane progressive waves – wave equation - Physics of light-Measurement of light and its unit – an overview of limits of vision and colour vision - Physics of sound, Normal sound levels –ultrasound fundamentals – Generation of ultrasound (Ultrasound Transducer) Non-destructive Testing – pulse echo system through transmission and reflection modes - A,B and C – scan displays, Medical applications – Sonogram.

UNIT V - NUCLEAR AND PARTICLE PHYSICS 9

Radioactivity - characteristics of radioactive material – isotopes - probing by isotopes, reactions involved in the preparation of radioisotopes, the Szilard-Chalmer's reaction – radiochemical principles in the use of tracers - nuclear medicines – Interaction of charged particles with matter –Specific ionization, Linear energy transfer, range, Bremsstrahlung, Annihilation. Gamma-Ray Spectrometry- Liquid Scintillation Counters-Characteristics of Counting Systems-Gamma Well Counters

TEXT BOOKS:

1. Kasap, S.O. "Principles of Electronic Materials and Devices", McGraw-Hill Education, 2007.
2. Umesh K Mishra &Jasprit Singh, "Semiconductor Device Physics and Design", Springer, 2008.
3. Wahab, M.A. "Solid State Physics: Structure and Properties of Materials". Narosa Publishing

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1. S. O. Pillai, Solid state physics, New Age International, 2015
2. Arthur Besier and S. RaiChoudhury,Concepts of Modern Physics (SIE), 7th edition, McGraw-Hill Education, 1994.
3. J.B.Rajam, Atomic Physics, 7th edition, S.Chand, 2010.
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UNIT - I

PROPERTIES OF MATTER

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UNIT - I

PROPERTIES OF MATTER

1.1. INTRODUCTION

Elasticity is the branch of physics which deals with the properties of the solid body. As an engineer must have sound knowledge about the elastic properties of a rigid body which helps to predict the behaviour of rigid body under forces applied. This will help to design tools and new instruments in all engineering field. In order to learn about the elasticity, let us consider a solid body. A solid body is composed of a great number of molecules or atoms arranged in a particular fashion. Each molecule is acted upon by forces due to the neigbouring molecules. If the distance between any two points in a solid body is invariable, the body is said to be a rigid body. In practice it is not possible to have a perfect rigid body. All bodies get deformed under the action of force, some more and some less.

Even a quartz fibre which is the nearest approach to a perfectly elastic body does not regain its original size and shape from very large deformations. Similarly putty which is the nearest approach to a perfectly plastic body tends to regain from small deformations. Thus, a body is said to be elastic or plastic when compared to another. Bodies which are homogeneous and isotropic are considered to understand the elasticity.

When we apply a force on a solid object, it may deform (Change in shape/size). The property by which the body regains its original shape and size, on removal of the deforming force is called elasticity.

If a body resists deformation and recovers its original size or volume completely and immediately on the removal of the deforming forces it is called ***perfectly elastic body.***

If it does not completely retain its altered shape and size, it is said to be ***perfectly plastic.***

In general bodies are in between these two extreme limits. There is no perfectly elastic (or) perfectly plastic body.

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1.2. STRESS AND STRAIN

When there is a load on the body, the forces of reaction come into play internally in it, tending to restore it to its original condition. This restoring or recovering force per unit area set up inside the body is called ***stress.*** The unit of stress is pascal or Nm^{-2}.

$$Stress = \frac{Force}{Area}$$

The change produced in the body due to change in dimension of a body under a system of forces. Thus amount of deformation suffered by a body under applied external forces. ***The strain is defined as change in dimension to original dimension.***

$$\text{Strain}=\frac{\text{change in dimension}}{\text{original dimension}}$$

Strain has no unit.

There are three types of strain; they are explained in detail as follows:

1. **Longitudinal or Tensile Strain**
   The change in length per unit original length without any change in shape is called as longitudinal strain.

2. **Shear or Shearing Strain**
   When tangential forces act on a body, it undergoes a change in shape. The angular deformation produced is called shearing strain or shear.

3. **Volumetric Strain**
   It is defined as the change in volume per unit original volume, without any change in shape.

1.3. HOOKE'S LAW

According to this law the stress is proportional to strain producing it within elastic limit

$$\text{Stress } \alpha \text{ strain}$$

$$\text{Or } \frac{Stress}{Strain} = \text{a constant} = E$$

The constant E is called the co-efficient of Elasticity or modulus of Elasticity. The value of this constant depends upon the nature of the material. The maximum value of the stress within a body completely regains its original condition of shape and size when the deforming forces are removed is known as the ***elastic limit***.

It has the same unit of stress that is Nm^{-2} Or pascal

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1.4. STRESS – STRAIN DIAGRAM (ELASTIC BEHAVIOUR OF A MATERIAL)

The elastic behaviour of a material of ductile wire such as copper or low carbon steel can be studied by plotting a curve between the stress along the y axis and the corresponding strain on the x axis. The curve is called stress-strain curve. Let a ductile wire be clamped at one end and loaded at the other end gradually from zero value until the wire breaks down. The nature of the stress-strain curve for low carbon steel wire is shown in Fig. 1.1.

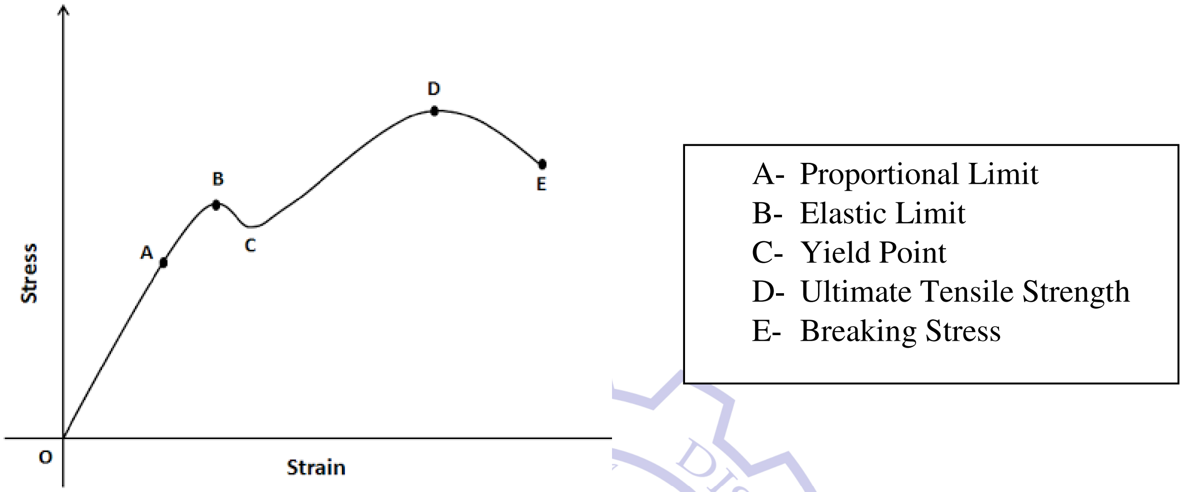

Fig. 1.1 Stress-Strain curve for ductile materials

1. The part OA of the curve is a straight line which shows that up to the point A, the stress is proportional to the strain i.e Hooke's law is obeyed. The point A is called ***proportional limit*** which is measured by the maximum stress that can be developed in the given material without causing a deviation from Hooke's law.
2. Beyond A up to B even for the smallest increase in stress larger strain is produced in the body. B is called ***Elastic Limit*** i.e up to the point B, the wire behaves as a perfectly elastic body. However, it should be remembered that it is not necessary for the part AB of the curve that the stress should be proportional to the stain. These two points A and B are very nearer to each other and may coincide for some materials.
3. If the wire is loaded beyond the elastic limit elongation is permanent i.e there is permanent deformation in the body after the removal of the deforming forces. Further loading up to C and this point is called ***Yield point*** at which extension of the wire increases rapidly without an increase in the load. The value of the stress at the yield point is called yield strength of the material.
4. The wire is further loaded, a point represented by D is reached after which the wire begins to neck down or flow locally so that it's cross sectional area no longer remains uniform. At this point D, the wire begins to thin at some point where it finally breaks. At the point D, the value of the developed stress is maximum and is called the ***ultimate tensile strength or tensile strength*** of the given material. Hence, tensile strength is defined as the maximum value of stress withstand by the material before fracture under the steady load.

$$Tensile\ strength\ = \frac{Maximum\ tensile\ load}{Original\ cross-sectional\ area}$$

5. The stress corresponding to the point E where the wire actually breaks down, is called ***breaking stress***. The value of the breaking stress is no practical importance whereas the position of point D is very useful in knowing the ultimate tensile strength of the material.

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Substances like quartz, bronze and silver can regain their original condition immediately on removal of the deforming forces. That is why they are frequently employed as the suspensions in galvanometers and electrometers, etc., but some other materials, like glass fibers takes hour to recover from strain. This delay in regaining the normal condition is called elastic after effect.

Normally the working stress on a body is kept far below the ultimate tensile stress and is never allowed to cross the elastic limit. The above fact is practiced by all design engineers to get higher stability and reliability of the structures. The ration between the ultimate tensile stress and the working stress is called the ***safety factor***.

$$Safety\ factor = \frac{Ultimate\ tensile\ stress}{working\ stress}$$

Working load or working stress is determined by the designer on the basis of his experience and knowledge. Thus the safety factor depends upon the engineering materials and the standard of workmanship.

Uses of stress-strain diagram

1. It is used to categorize the materials into ductile or brittle or plastic in nature.
2. It provides engineers and designers a graphical measure of the strength and elasticity of the material.

1.5. TYPES OF MODULUS

Corresponding to the three types of strain, there are three kinds of moduli of elasticity. They are Young's modulus (E), Bulk modulus (K) and Rigidity modulus (n). Modulus is defined as ratio between stress and strain.

YOUNG'S MODULUS (E)

The young's modulus (E) is defined as ratio of linear or longitudinal stress to linear or longitudinal strain.

$$E = \frac{longitudinal\ stress}{longitidinal\ strain}$$

Consider a rod of original length 'L' and the cross section area be A

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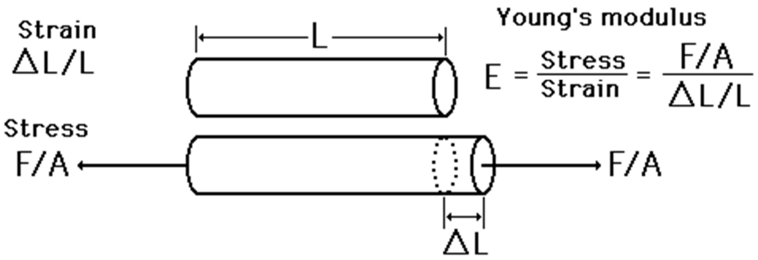

Fig. 1.2 Young's Modulus

$$\text{The longitudinal stress } = \frac{Force}{Area} = \frac{F}{A}$$

$$\text{The longitudinal strain} = \frac{Change\ in\ length}{Original\ length} = \frac{\Delta L}{L}$$

Then Young's modulus

$$E = \frac{F/A}{l/L} = \frac{FL}{Al}$$

Unit of Young's modulus is Nm^{-2} (or) pascal

BULK MODULUS (K)

When uniform force is applied normally to the surface of the entire body, it produces a change in volume without any change of shape. The force applied per unit area or pressure gives the bulk stress. The change in volume per unit volume gives the bulk strain.

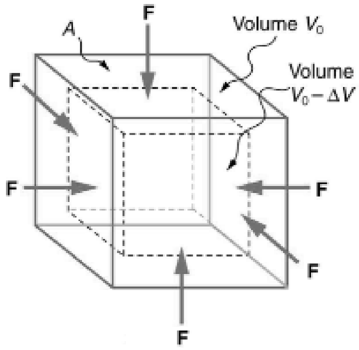

Fig.1.3 Bulk Modulus

Bulk stress = $\frac{Change\ in\ normal\ force}{Area}$ = Δp

Δp – Change in normal pressure

Bulk Strain = $\frac{Change\ in\ Volume}{Original\ Volume} = \frac{\Delta v}{V}$

Bulk Modulus (K) = $\frac{-\Delta p}{\frac{\Delta v}{V}}$

The negative sign indicates when the pressure applied increases, the volume decreases.

RIGIDITY MODULUS (n)

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The ratio of tangential force per unit area to the angular deformation produced is called modulus of rigidity.

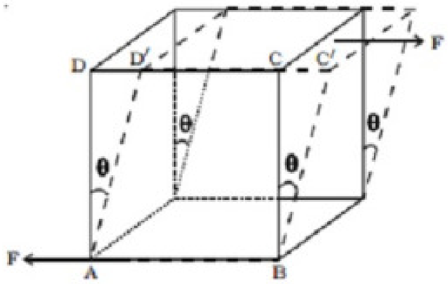

Fig.1.4 Rigidity modulus

Modulus of rigidity $n = \frac{Tangential\ stress}{Shear\ Strain} = \frac{T}{\theta}$

In general $\theta = \tan\theta = AA' / BA$

$$\theta = AA' / BA = \frac{l}{L}$$

$$n = \frac{F/a}{l/L} = \frac{FL}{al}$$

The unit of rigidity modulus of elasticity is Nm^{-2}

1.6 POISSON'S RATIO

When a wire is stretched by means of a force, it is elongated. It is observed that along with an increase in length of wire, a corresponding contraction in its diameter also takes place. The ratio of the change in diameter to the initial diameter is known as lateral strain. Within the elastic limit the ratio of the lateral strain to the longitudinal strain is constant for the material of the body and is known as poisson's ratio. It is denoted by (σ).

$$Poisson's\ ratio = \frac{Lateral\ strain}{Longitudianl\ strain}$$

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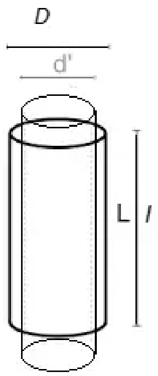

Fig.1.5 Poisson's ratio

Let us consider a wire, fixed at one end and is stetched along the other end as shown in the Fig.1.5. After the application of force, the length increases from L to L+*l* and the corresponding diameter decreases from D to d, then

$$Lateral\ strain = \frac{l}{L}$$

and

$$Longitudianl\ strain = \frac{(D-d)}{D}$$

Therefore,

$$\sigma = \frac{Lateral\ Strain(\beta)}{longitudinal\ strain(\alpha)} = \frac{(D-d)/D}{l/L}$$

$$\sigma = \frac{-L\,(D-d)}{Dl}$$

The negative sign indicates that longitudinal and lateral strains are in opposite sense.

Most materials have positive Poisson's ratio values ranging between 0.0 and 0.5. Certain materials with a negative Poisson's ratio have been called anti-rubber, dilational or auxetic materials. When stretched, the auxetic materials become thicker perpendicular to the applied force. This occurs due to their internal structure and the way this deforms when the sample is uniaxially loaded. Auxetics may be ussful in applications such as body armor, packing material, knee and elpow pads, robust shock absorbing material and sponge mops.

1.7 FACTORS AFFECTING ELASTICITY

Apart from elastic fatigue, some materials will have change in their elastic property because of the following factors:

1. Effect of temperature
2. Effect of stress
3. Effect of annealing
4. Effect of impurities
5. Nature of crystals

Let us discuss in details in the following manner.

1. **Effect of temperature**

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Normally, the elastic property increases with decrease in temperature and decreases with increase in temperature. Examples: A carbon filament which is highly elastic at normal temperature becomes plastic when it is at high temperature. Lead is not a good elastic material. But at low temperature it becomes a very good elastic material. Invar is a special alloy used for making pendulums and its elasticity is not affected by the temperature change in the normal temperature range.

2. Effect of Stress

When a material is subjected to a large number of cycles of stresses, it loses its elastic property even within elastic limit, it results in the formation of large crystal grain, which reduces the elastic property of the material. Taking these into account, the working stress on an engineering piece is kept far below its ultimate tensile strength.

3. Effect of annealing

In annealing the material is subjected to heat at very high temperatures and then is allowed to cool slowly which is adopted to increase softness and ductility in the materials. But it decreases the elastic the elastic properties of the material by decreasing the tensile strength and yield point of the material.

4. Effect of Impurities

The elastic properties of material may increase or decrease depends upon the impurities added with the materials.

Examples:

1. When potassium is added with gold, the elastic property of gold increases
2. When carbon is added to molten iron , the elastic property of iron decreases

These kinds of impurities atoms generally have different atomic radii and electronic structures which decrease the elastic properties of the base metal.

5. Nature of the Crystals

The elasticity also depends on the type of the crystals. For single crystal, the elasticity is more and poly crystal the elasticity is less.

1.8 COUPLE

Couple refers to two parallel forces that are equal in magnitude, opposite in direction and do not share a line of action.

Force couple or **pure moment**. Its effect is to create rotation without translation, or more generally without any acceleration of the centre of mass. In rigid body mechanics, force couples are *free vectors*, meaning their effects on a body are independent of the point of application.

Why are a door's doorknob and hinges placed near opposite edges of the door?

This question actually has an answer based on common sense ideas. The harder we push against the door and the farther we are from the hinges, the more likely we are to open or close the door. When a force is exerted on a rigid object pivoted about an axis, the object tends to rotate about that axis. The tendency of a force to rotate an object about some axis is measured by a vector quantity called torque τ (tau).

A couple is a pair of forces, equal in magnitude, oppositely directed, and displaced by perpendicular distance or moment.

This is called a *"simple couple"*. The forces have a turning effect or moment called a torque about an axis which is normal (perpendicular) to the plane of the forces. The SI unit for the torque of the couple is newton metre.

If the two forces are **F** and **–F**, then the magnitude of the torque is given by the following formula:

$$\tau = F\,d$$

where

τ is the moment of couple
F is the magnitude of one of the forces
d is the perpendicular distance between the forces, sometimes called the *arm* of the couple

Torsion of circular shafts

Definition of Torsion: Consider a shaft rigidly clamped at one end and twisted at the other end by a torque **T = F X d** applied in a plane perpendicular to the axis of the bar such a shaft is said to be in torsion.

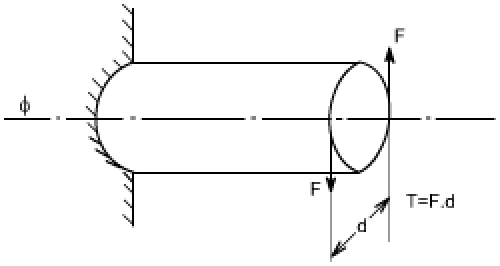

Fig 1.6 a Torsion of shaft

Effects of Torsion: The effects of a torsional load applied to a bar are

(i) To impart an angular displacement of one end cross – section with respect to the other end.

(ii) To setup shear stresses on any cross section of the bar perpendicular to its axis.

Shaft**:** The shafts are the machine elements which are used to transmit power in machines.

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Twisting Moment: The twisting moment for any section along the bar / shaft is defined to be the algebraic sum of the moments of the applied couples that lie to one side of the section under consideration. The choice of the side in any case is of course arbitrary.

Shearing Strain: If a generator a – b is marked on the surface of the unloaded bar, then after the twisting moment 'T' has been applied this line moves to ab'. The angle 'g' measured in radians, between the final and original positions of the generators is defined as the shearing strain at the surface of the bar or shaft.

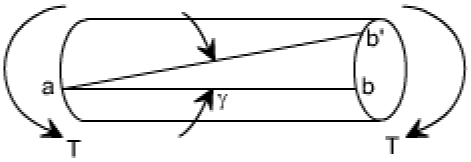
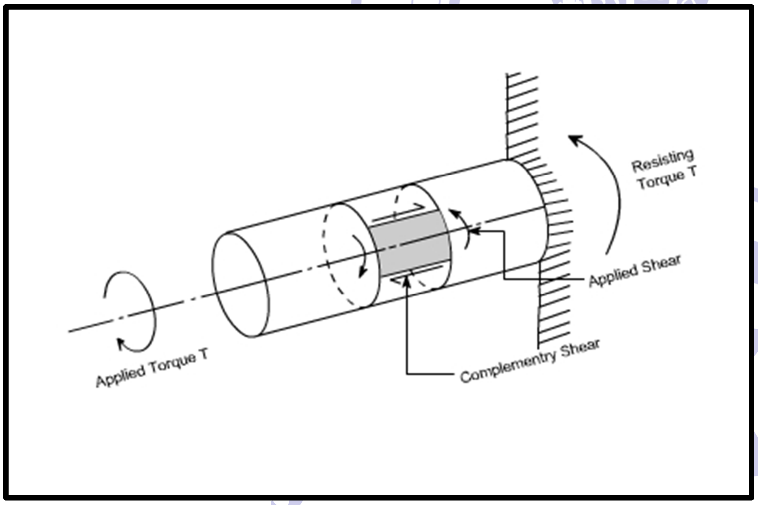


Fig 1.7 b shearing stress



Modulus of Elasticity in shear: The ratio of the shear stress to the shear strain is called the modulus of elasticity in shear OR Modulus of Rigidity and in represented by

$$G = \frac{\tau}{r}$$

Angle of Twist: If a shaft of length L is subjected to a constant twisting moment T along its length, than the angle q through which one end of the bar will twist relative to the other is known is the angle of twist.

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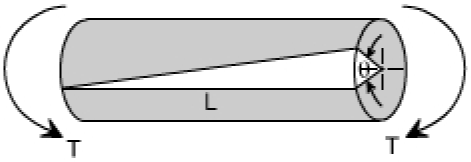

Fig 1.8 Angle of twist

For the purpose of designing a circular shaft to withstand a given torque, we must develop an equation giving the relation between twisting moment, maximum shear stress produced, and a quantity representing the size and shape of the cross-sectional area of the shaft.

Refer to the figure 1.9d shown where a uniform circular shaft is subjected to a torque it can be shown that every section of the shaft is subjected to a state of pure shear, the moment of resistance developed by the shear stresses being everywhere equal to the magnitude, and opposite in sense, to the applied torque. For the purpose of deriving a simple theory to describe the behavior of shafts subjected to torque it is necessary make the following base assumptions.

Assumption

(i) The material is homogenous i.e of uniform elastic properties exists throughout the material.
(ii) The material is elastic, follows Hook's law, with shear stress proportional to shear strain.
(iii) The stress does not exceed the elastic limit.
(iv) The circular section remains circular
(v) Cross section remain plane.
(vi) Cross section rotate as if rigid i.e. every diameter rotates through the same angle.

1.9 -TWISTING COUPLE ON A WIRE – EXPRESSION FOR TORQUE PER UNIT TWIST

Consider now the solid circular shaft of radius R subjected to a torque T at one end, the other end being fixed Under the action of this torque a radial line at the free end of the shaft twists through an angle θ , point A moves to B, and AB subtends an angle ' γ ' at the fixed end. This is then the angle of distortion of the shaft i.e the shear strain.

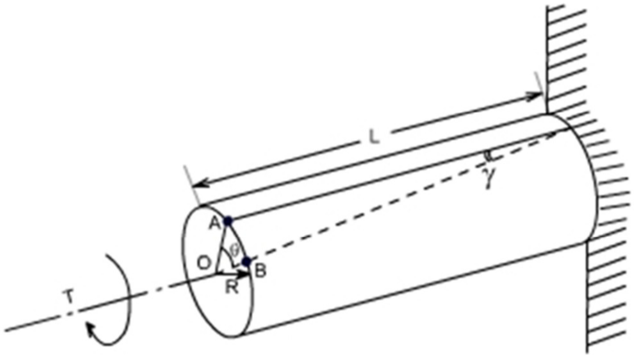

Fig 1.9 a Twisting couple

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Since angle in radius = arc / Radius

$$\text{arc } AB = r\,\theta = L\,\gamma \qquad [\text{since } L \text{ and } \gamma \text{ also constitute the arc } AB]$$

Thus, $\gamma = r\,\theta / L$

From the definition of Modulus of rigidity or Modulus of elasticity in shear

Rigidity Modulus $n = \frac{Tangential\ Stress}{Shearing\ Strain}$

Therefore, Shearing Stress = n γ

Substituting for γ, Shearing Stress = $\frac{n\,r\,\theta}{L}$

Also, Shearing Stress = $\frac{Shearing\ Force}{Area}$

Shearing Force = Shearing Stress X Area on which the Shearing force is acting

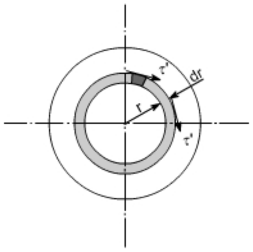

Fig 1.10b cross sectional view of twisting couple

$$\text{Shearing Force (F)} = \frac{n\,r\,\theta}{L} \times 2\pi\, r.dr$$

Moment of force about the axis of the cylinder = Shearing force x distance (r)

$$= F \times r$$

$$= \frac{n\,r\,\theta}{L} .\, 2\pi\, r.dr.r$$

$$\text{Moment of force} = \frac{2\pi n\theta r^3}{L} dr$$

For the Twisting couple of the whole wire

$$C = \int_0^R \frac{2\pi n\theta r^3}{L} dr$$

$$C = \frac{\pi n\theta R^4}{2L}$$

If θ is unity i.e. θ = 1 radian

The torques per unit twist $C = \frac{n\,\pi\,R^4}{2\,L}$

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1.10 Torsional Stress & deformation (Theory)

A torsion pendulum is shown in figure and it consist of a cradle ‘D’ from a fixed torsion head ‘H’ by means of a wire ‘w’

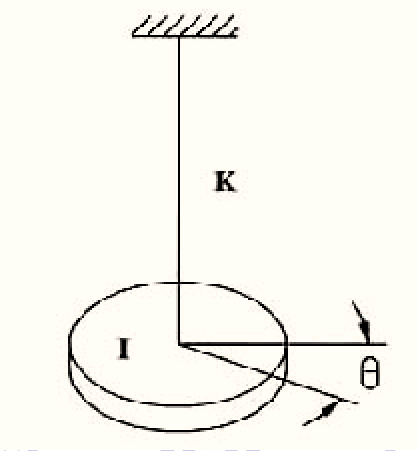

Fig.1.10 Torsional Pendulum

When the disc is rotated in a horizontal plane so as to twist the wire, the various elements of the wire undergo shearing strains and the restoring couple produces the oscillations.

If the angle of twist at the lower end of the wire is θ, then the restoring couple is Cθ, where, C - torsional rigidity.

This couple acting on the disc produces in it an angular displacement, therefore from the law of conservation of energy, the energy of the system is conserved:

Total energy of the Torsional pendulum = Potential Energy + Kinetic Energy -------- (1)

Then the restoring couple through an angle θ $= \int_0^Q moment\ of\ couple\ X\ d\theta$

$$= \int_0^Q C\theta . d\,\theta$$

Potential Energy (PE) = $C\theta^2 / 2$ ------------------------- (2)

The kinetic energy of the disc (KE) = $\frac{1}{2} I\, \omega^2$ ----------------------- (3)

Then total energy (TE) $= C\theta^2 / 2 + \frac{1}{2} I\, \omega^2$ = Constant ------------ (4)

Differentiating equation (4) with respect to time

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$$C\theta\frac{d\theta}{dt} + I\omega\frac{d\omega}{dt} = 0$$

Since the angular velocity $\omega = \frac{d\theta}{dt}$ and an angular acceleration $\frac{d\omega}{dt} = d^2\theta / dt^2$

We can write

$$C\theta\frac{d\theta}{dt} + I\frac{d\theta}{dt}\, d^2\theta / dt^2 = 0$$

$$\frac{d\theta}{dt}\,[\, C\theta + I\, d^2\theta / dt^2 \,] = 0$$

Where $\frac{d\theta}{dt} \neq 0$

$$C\theta + I\, d^2\theta / dt^2 = 0$$

$$d^2\theta / dt^2 = -\frac{C}{I}\theta \quad \text{------------------------ (5)}$$

angular acceleration $= -\frac{C}{I}\theta$ ------------------------ (6)

(-) Negative sign indicates angular acceleration in opposite direction.

Torsional oscillation belongs to simple harmonic motion,
Hence the period of oscillation

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$T = 2\pi\sqrt{\frac{Displacement}{accleration}}$$

$$T = 2\pi\sqrt{\frac{\theta}{\left(\frac{C}{I}\right)\theta}}$$

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{---------------------- (7)}$$

C - twisting couple

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Twisting couple $C = \frac{\pi n \theta r^4}{2l}$ ----------------------- (8)

Substituting the equation (8) in equation (7)

$$T = 2\pi \sqrt{\frac{I}{\frac{\pi n r^4}{2l}}}$$

$$T = 2\pi \sqrt{\frac{2l\,I}{\pi n r^4}} \quad \text{----------------------- (9)}$$

Squaring the equation (9) on both sides

$$T^2 = 4\pi^2 \frac{2lI}{n\pi r^4}$$

$$n = \frac{8\pi I l}{\pi r^4 t^2} \quad \text{Nm}^{-2} \quad \text{----------------------------- (10)}$$

I – Moment of Inertia

1.10.1 MOMENT OF INERITA OF A REGULAR AND IRREGULAR BODY USING TORSION PENDULUM

Regular Object

The moment of Inertia of the disc and the rigidity modulus of the wire using torsional pendulum could be determined by the following

(i) The disc is set into oscillations without any mass and the time period of oscillations is measured.

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$T^2 = \frac{4\pi^2 I}{C} \quad \text{-------------(1)}$$

Where,

I = Moment of Inertia of the disc about the axis of the rotation

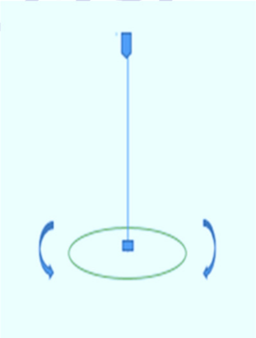

Fig1.11a Torsional Pendulum without mass

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C = restoring couple.

(ii) When two equal cylindrical masses are placed over the disc at equal distances (d1) from the centre of the disc and is set into torsional oscillations.

Time period of oscillation T_1 is measured.

$T_1 = 2\pi\sqrt{\frac{I_1}{C}}$ or $T_1^2 = \frac{4\pi^2 I_1}{C}$ ----------------(2)

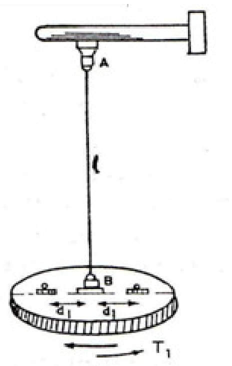


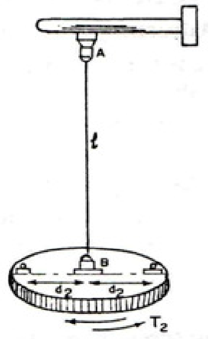
Fig 1.11b Torsional Pendulum with mass at distance d_1

Where, I_1 is the moment of inertia of the disc along with the cylindrical masses placed over the disc at a distance d_1

(iii) When the two cylindrical weights are placed at the extreme ends of the disc and the distance d_2 from the centre of the one of the weight and the point of suspension wire is noted. The disc is now subjected to torsional oscillations.

$T_2 = 2\pi\sqrt{\frac{I_2}{C}}$

Fig1.11c Torsional Pendulum with mass at distance d_2



$T_2^2 = \frac{4\pi^2 I_2}{C}$ ---------------------(3)

Where I_2 is the moment of Inertia of the disc along with two equal masses at a distance d_2.

a) Moment of Inertia of the disc

From eqns (1), (2) and (3), we can write

$$\frac{T^2}{T_2^2 - T_1^2} = \frac{4\pi^2 I/C}{\frac{4\pi^2}{C}(I_2 - I_1)}$$

$$\frac{T^2}{T_2^2 - T_1^2} = \frac{I}{I_2 - I_1}$$

$I = \frac{(I_2 - I_1)T^2}{T_2^2 - T_1^2}$ -------------(4)

from the parallel axis theorem , the moment of Inertia I_1

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$$I_1 = I + 2\,I_m + 2\,md_1^2 \text{ ----------------(5)}$$

For $$I_2 = I + 2\,I_m + 2\,md_2^2 \text{ ------------------(6)}$$

From eqn (5) and (6)

$$I_2 - I_1 = 2\,m\,(d_2^2 - d_1^2) \text{--------(7)}$$

Moment of Inertia of the disc about the axis of the rotation

$$I = \frac{2m(d_2^2 - d_1^2)T^2}{T_2^2 - T_1^2} \text{ ----------------(8)}$$

(b) Rigidity modulus of the wire

The rigidity of the modulus of the wire

$$\eta = \frac{8\pi I l}{T^2 r^4} \text{ ---------------(9)}$$

Substituting Eqn (8) in (9) , we have

$$\eta = \frac{8\pi l}{T^2 r^4}\,\frac{2m(d_2^2 - d_1^2)T^2}{T_2^2 - T_1^2} \text{ ---------(10)}$$

(ii) Irregular Object

For the determination of moment of inertia for any irregular object , torsion pendulum provided with cradle fixed to a rectangular metallic frame where any irregular object could be placed.

Time period of oscillations were noted when the cradle is set into oscillations.

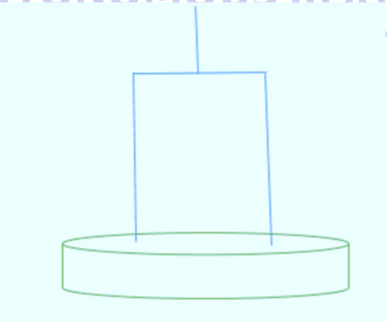

Fig. 1.12 Irregular body

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$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$T^2 = \frac{4\pi^2 I}{C} \text{ --------------(1)}$$

I = moment of Inertia of the cradle.

Now the time period of oscillations were noted for the known object.

$$T_1 = 2\pi\sqrt{\frac{I+I_1}{C}}$$

$$T_1^2 = \frac{4\pi^2}{C^2}(I + I_1) \text{ -------------(2)}$$

Now, the regular body is removed from the cradle and an irregular body is placed over the cradle and is allowed to produce torsion oscillations.

$$T_2 = 2\pi\sqrt{\frac{I + I_2}{C}}$$

$$T_2^2 = \frac{4\pi^2}{C^2}(I + I_2) \text{ ---------(3)}$$

where I_2 is the moment of inertia of an irregular body ,

Therefore for an unknown object , the moment of Inertia can be calculated from eqn(1),(2) &(3)

$$I_2 = I_1\left(\frac{T_2^2 - T^2}{T_1^2 - T^2}\right)$$

1.11 SIMPLE BENDING THEORY

When a beam having an arbitrary cross section is subjected to a transverse loads the beam will bend. In addition to bending the other effects such as twisting and buckling may occur, and to investigate a problem that includes all the combined effects of bending, twisting and buckling could become a complicated one. Thus we are interested to investigate the bending effects alone, in order to do so, we have to put certain constraints on the geometry of the beam and the manner of loading.

Assumptions:

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The constraints put on the geometry would form the **assumptions:**

1. Beam is initially **straight** , and has a **constant cross-section.**

2. Beam is made of **homogeneous material** and the beam has a **longitudinal plane of symmetry.**

3. Resultant of the applied loads lies in the plane of symmetry.

4. The geometry of the overall member is such that bending and not buckling is the primary cause of failure.

5. Elastic limit is nowhere exceeded and **'E'** is same in tension and compression.

6. Plane cross - sections remains plane before and after bending.

1.12 BENDING MOMENT

A beam is defined as rod of uniform cross-section whose length is much greater as compared to its other dimensions, so that shearing stresses over any section of it are negligibly small. They are usually set in horizontal position and are designed to support heavy loads. They are used in building to support roofs and in bridges to support the load of vehicles passing over them.

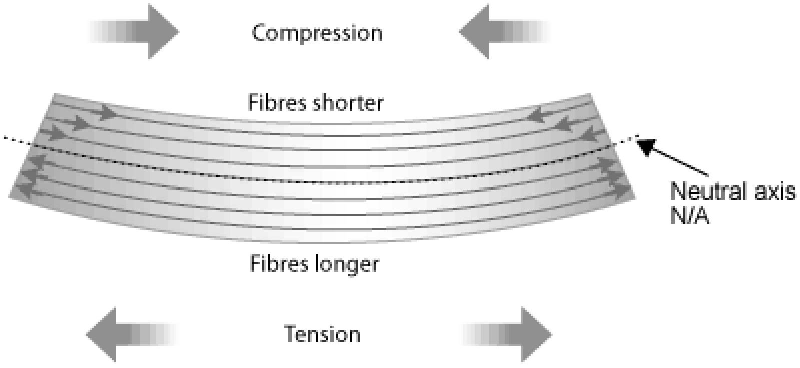

Fig.1.12 A bending of a beam

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Let us consider a beam of uniform rectangular cross section as shown in the Fig.1.12. A beam may be assumed to consist of a number of parallel longitudinal metallic fibers placed one over the other and are called as filaments. Under the bent condition, the convex side of the beam is extended while those on the concave side are compressed. In between these filaments, this is neither lengthened nor shortened, but remains constant in length. This filament is called **neutral filament** and the axis of the beam lying on the neutral filament is called the **neutral axis**. Further the deformation of any filament can be measured with reference to neutral axis.

1.12.1 DERIVATION OF BENDING MOMENT OF BEAM

Let us consider the section abcd in Fig.1.13 represents a small longitudinal bent beam. Let PQ represent a filament at the neutral surface and P'Q' another filament at distance y from PQ. In a bent beam, PQ and P'Q' are arcs of a circle having their centre at O and of radii R and R + y respectively. If θ is the angle for POQ,

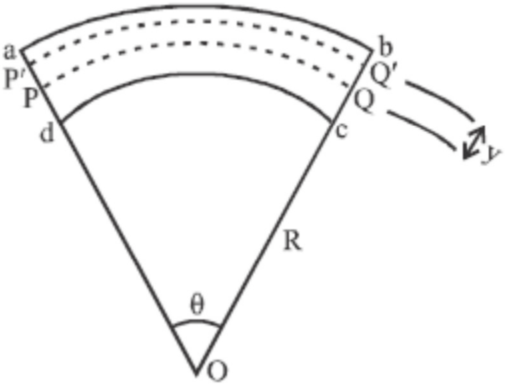

Fig. 1.13 Bending moment of a beam

we have

$$PQ = R\,\theta$$

and $$P'Q' = (R+y)\,\theta$$

Change in length produced in the filament PQ due to bending $= P'Q' - PQ$

$= (R + y)\theta - R\theta$

$= R\theta + y\theta - R\theta$

$= y\theta$

The extensional strain on the filament $= \dfrac{\text{change in the length}}{\text{original length}}$

$$= \frac{Y\theta}{R\theta}$$

$$= \frac{Y}{R}$$

Since the beam is not subjected to either shearing or bulk strain the extensional or compressional strains on the filaments are due to forces acting along the length of the filaments.
If Y is the Young's modulus of the material of the beam the longitudinal stress on the filament P' Q'.

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$$= Y \times \text{Strain}$$

$$= Y \times \frac{y}{R}$$

Let a be the area of cross section of the filament. The longitudinal force on the filament is stress × area

$$\text{Longitudinal force} \quad = \frac{y}{R} \times a$$

$$= \frac{Yy}{R} a$$

The moment of the longitudinal force about the neutral axis = Force × distance

$$= \frac{Yy}{R} a \times y$$

$$= \frac{Y}{R} a\, y^2$$

As the moment of the forces acting on both the upper and lower halves of the section are in the same direction, the total moment of forces acting on the filaments due to straining

$$= \sum \frac{Y}{R} a\, y^2$$

$$= \frac{Y}{R} \sum ay^2$$

Here, the quantity $\sum ay^2$ is the geometrical moment of inertia and is equal to AK^2, A being the total area of the section and K being the radius of gyration of the beam. The quantity $\sum$ ay2 is represented as Ig where Ig is the geometrical moment of inertia of the bar. The total moment of forces acting on all the filaments is called bending moment.

$$\text{Bending moment} = \frac{YAK^2}{R}$$

$$\text{Bending moment of the beam} = \frac{Y}{R} I_g$$

Case (i)

For rectangular cross section

If b is the breadth and d is the thickness of the beam, then

$$\text{Area } A = bd \text{ and } K^2 = \frac{d^2}{12}$$

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$$Ig = AK^2 = \frac{\mathrm{bd.d}^2}{12} = \frac{bd^3}{12}$$

Substituting the value of Ig in the bending moment of a beam equation

$$\text{Bending moment for a rectangular cross section} = \frac{\mathrm{Ybd}^3}{12\mathrm{R}}$$

Case (ii)

For a circular cross section if r is the radius, then area A = πr^2

And $\quad K^2 = \frac{r^2}{4}$

$$Ig = AK^2 = \frac{\pi r^2 \times r^2}{4}$$

$$Ig = \frac{\pi r^4}{4}$$

Substituting value of Ig in the bending of a beam equation, then the equation becomes

$$\text{Bending moment for a circular cross section} = \frac{\pi \mathrm{Yr}^4}{4\mathrm{R}}$$

1.13 CANTILEVER

A beam clamped horizontally at one end and loaded with a weight at the free end is called a **cantilever**.

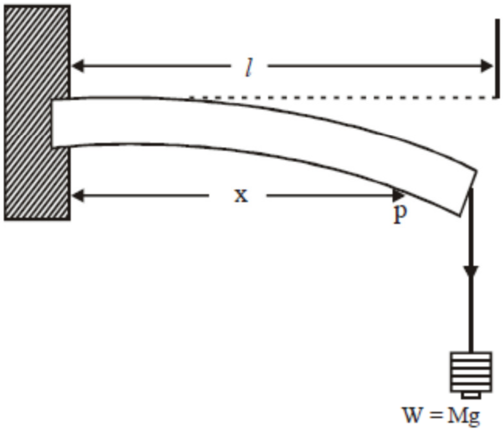

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Fig. 1.14 Cantilever

Consider a section at P at a distance x from the fixed end. Neglecting the weight of the part between P and the free end, the moment of the external couple = Mg(l - x).

The bending moment of the section at P = $\frac{YAK^2}{R}$, where R is the radius of curvature of the neutral axis at P

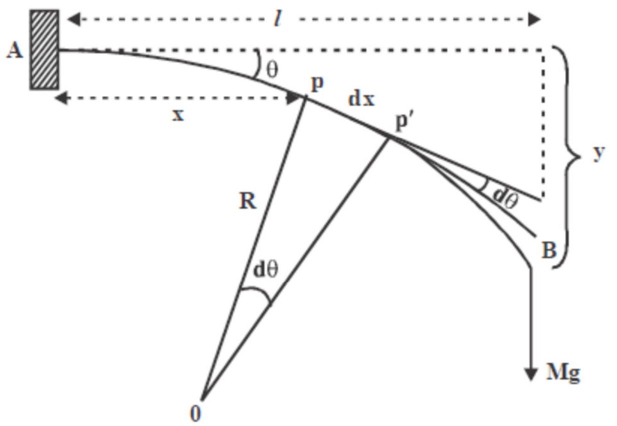

Fig. 1.14a Depression at the free end of cantilever

$$\text{At equilibrium, } Mg(l - x) = \frac{YAK^2}{R}$$

$$R = \frac{YAK^2}{Mg(l-x)} --------(1)$$

From this expression we find that as x increases, R also increases, the maximum and minimum values being at the free and fixed ends respectively. Since the radius of curvature of the neutral axis (R) varies from point to point of the cantilever (x), the bending is said to be non-uniform.

Let P’ be another point on the bent cantilever at a small distance dx from P. Since P and P’ are very near we can assume that the radius of curvature R is practically the same and O is its centre of curvature.

Let the angle between the tangents at P and P’ be dθ.

Then angle POP’ = dθ

$$dx = Rd\theta$$

$$R = \frac{dx}{d\theta} --------(2)$$

Substituting the equation (2) in (1)

$$Mg(l-x) = YAK^2 \frac{d\theta}{dx}$$

$$d\theta = \frac{Mg(l-x)dx}{YAK^2} ----------(3)$$

If θ is the angle between the tangents at A and B, we have

$$\theta = \int_0^l \frac{Mg(l-x)dx}{YAK^2}$$

$$\theta = \frac{Mgl}{YAK^2}\int_0^l dx - \frac{Mg}{YAK^2}\int_0^l x dx$$

$$= \frac{Mgl^2}{YAK^2} - \frac{Mgl^2}{2YAK^2}$$

Deflection of cantilever $= \frac{Mgl^2}{2YAK^2} ----------(4)$

If dy is the depression of B due to the curvature at PP'

$$dy = (l-x)d\theta$$

$$d\theta = \frac{dy}{l-x} --------(5)$$

Substituting the equ.(5) in (3)

$$dy = \frac{Mg(l-x)^2 dx}{YAK^2}$$

The depression y is given by

$$Y = \int dy = \int_0^l \frac{Mg(l-x)^2 dx}{YAK^2}$$

$$= \frac{Mg}{YAK^2}\int_0^l (l-x)^2 dx$$

$$= \frac{Mg}{YAK^2}\left[l^2 x - \frac{2lx^2}{2} + \frac{x^3}{3}\right]_0^l$$

$$= \frac{Mg}{YAK^2}\left[l^3 - l^3 + \frac{l^3}{3}\right]$$

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$$\text{Depression of cantilever } y = \frac{Mgl^3}{3YAK^2}$$

Case i For rectangular cross section

If b is the breadth and d is the thickness of the beam then Ig becomes

$$Ig = \frac{bd^3}{12}$$

Substituting the value of Ig in the above (cantilever) equation

The depression produced at free end for a rectangular cross section

$$y = \frac{Mgl^3}{3Y\frac{bd^3}{12}}$$

$$y = \frac{4Mgl^3}{Ybd^3}$$

Case ii For circular cross section

If r is the radius of the circular cross section, then

$$Ig = \frac{\pi r^4}{4}$$

Substituting the value of Ig in the above (cantilever) equation

$$\textbf{Depression produced } \; y = \frac{Mgl^3}{3Y(\pi r^4/4)}$$

$$y = \frac{4Mgl^3}{3\pi Yr^4}$$

1.12.2 EXPERIMENTAL DETERMINATION OF YOUNG'S MODULUS BY CANTILEVER

A given beam one end is clamped by using a clamp on the table and free end is allowed to add load using weight hanger is as shown in the Fig. 1.15. A pin p is fixed at the free end of the beam by using wax. A travelling microscope is place in front of the whole setup for finding the position of the pin. Initially, the beam is brought into elastic mood by loading and unloading it several times. Now, with the weight hanger alone as the load, the travelling microscope is

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adjusted until its horizontal cross-wire coincides with the tip of the pin. The corresponding vernier scale reading is noted.

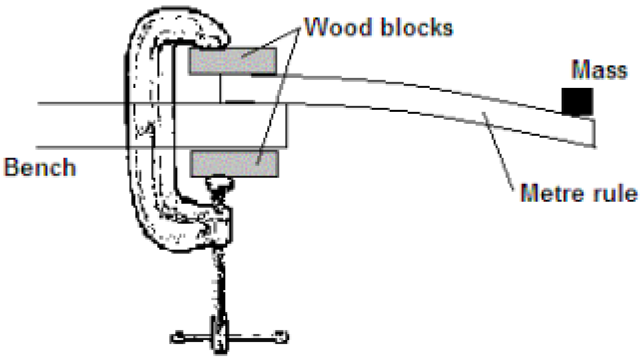

Fig. 1.15 Cantilever experiment

Taking the weight hanger as the dead load (W), the microscope is adjusted and the tip of the pin is made to coincide with the vertical cross wire. The reading is noted from the vertical scale of the microscope. Then weights are added (loading) to the weight hanger in steps of 50 gms upto to a total of 250 gms. In each case, it is ensured that the horizontal cross-wire coincides with the tip of the pin as in Fig.1.15 and the corresponding vertical scale readings are noted. The procedure is repeated by removing the weights (unloading) in the same steps of 50 gms each and the microscope readings are taken as before for the corresponding loads.

The mean of loading and unloading readings for each load M is recorded. The breadth b and the thickness t of the beam are measured by vernier caliper and screw gauge respectively. The observed values are substituted in the formula of Young's modulus to get the young's modulus of the material of the given beam by cantilever.

S.No	Load (gm)	Microscope Readings (m)		Average	Depression for a mass of m=50gm
		Loading	Unloading		
1	W			A	a ≈ b
2	W+50			B	b≈c
3	W+100			C	c≈d
4	W+150			D	d≈e
5	W+200			E	e≈f

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6	W+250	F	f≈g
7	W+300	G	
		Average y	

$$Young's\ Modulus\ Y = \frac{4Mgl^3}{3ybd^3}$$

By substituting all the values in the above equation we can calculate the Young’s modulus of the given beam.

1.13 UNIFORM BENDING

Let us consider a beam initially unstressed as shown in Fig.1.16. Now the beam is subjected to a constant bending moment (i.e. ‘Zero Shearing Force') along its length as would be obtained by applying equal couples at each end. The beam will bend to the radius R as shown in Fig. 1.17.

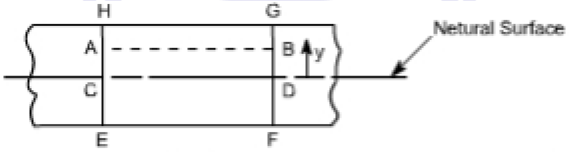

Fig.1.16 A view of a beam

As a result of this bending, the top fibers of the beam will be subjected to tension and the bottom to compression. It is reasonable to suppose, therefore, that somewhere between the two there are points at which the stress is zero. The locus of all such points is known as neutral axis. The radius of curvature R is then measured to this axis.

If the beam is loaded uniformly on its both ends, bending of the beam forms an arc of a circle. The elevation is produced in the beam. This type of bending is known as uniform bending.

In order to compute the value of bending stresses developed in a loaded beam, let us consider a beam AB supported symmetrically on two knife-edges at C and D and loaded with equal weights Mg at each end as shown in Fig.1.17.

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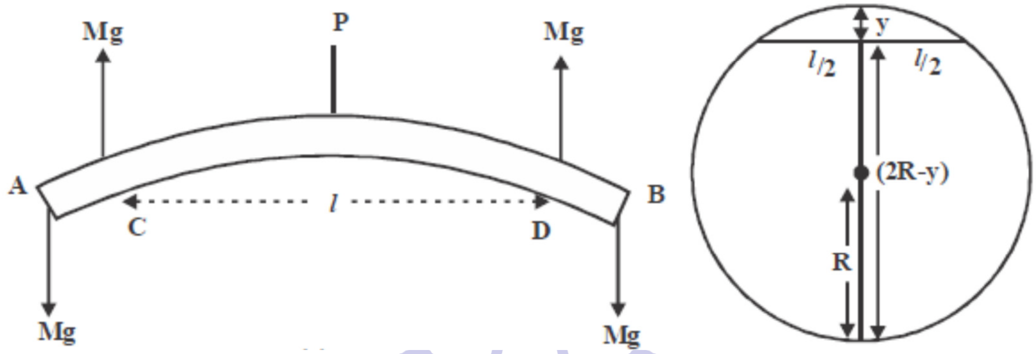

Fig. 1.17 Uniform bending method

The reaction at each knife-edge is each equal to Mg. Let P be mid portion of the beam. Consider the equilibrium of the portion PA of the beam. The only external forces on this beam are Mg acting vertically down at A and Mg vertically up at the knife-edge C. These constitute a couple of moment Mg × a, where a is the length of the beam projecting beyond the knife-edge.

The internal bending moment is = $\frac{YAK^2}{R}$

At equilibrium condition,

Internal bending moment = External bending moment

$$\frac{YAK^2}{R} = Mga -----------------(1)$$

Since Mga is a constant, R is also a constant. Therefore, the beam is bent into an arc of a circle of radius R. Hence the bending in this case is said to be uniform. If l is the distance between the knife-edges, and y is the elevation of the midpoint of the beam above the knife-edges, from the property of the circle, we have

$$(2R-y)y = \frac{l}{2} \times \frac{l}{2}$$

If y is small compared to R, then the term y^2 can be neglected.

$$2\,Ry = \frac{l^2}{4}$$

$$R = \frac{l^2}{8y}$$

Substituting the value of R, in equ.(1)

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$$\frac{YAK^2}{l^2/8y} = Mga$$

$$y = \frac{Mgl^2a}{8YAK^2}$$

$$Y = \frac{Mgl^2a}{8yAK^2}$$

But $AK^2 = \frac{bd^3}{12}$ for rectangular bar. Here b is the breadth and d is the thickness of the bar.

$$Y = \frac{Mgl^2a}{8y} \times \frac{12}{bd^3}$$

$$= \frac{3Mgl^2a}{2bd^3y}$$

$$Y = \frac{3Mgal^2}{2bd^3y}$$

1.13.1 DETERMINATION OF YOUNG MODULUS BY (UNIFORM BENDING) EXPERIMENT

The given metre scale is supported symmetrically on two knife edges C and D as shown in the Fig.1.18. Two equal weight hangers are suspended from the stirrups. A pin is vertically fixed at the centre of the scale. A travelling microscope is place in front of the whole setup for finding the position of the pin. Taking the weight hanger as the dead load (W), the microscope is adjusted and the tip of the pin is made to coincide with the vertical cross w ire. The reading is noted from the vertical scale of the microscope.

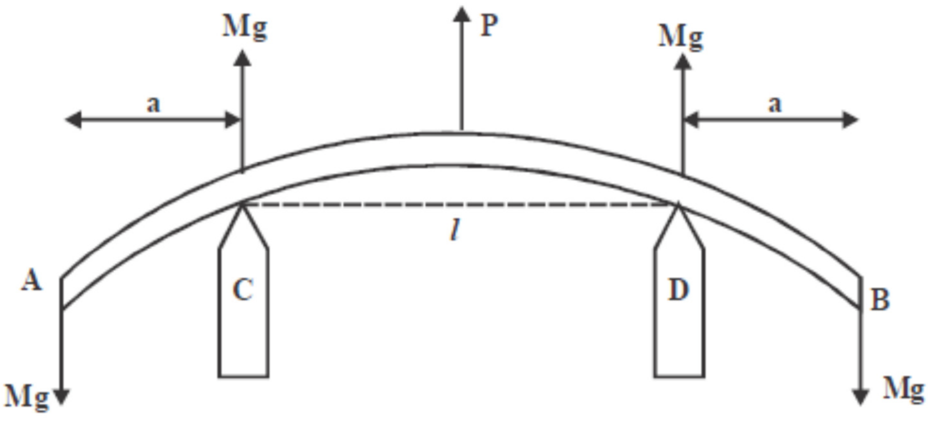

Fig. 1.18 Uniform bending experiment

Now the load on each hanger is increased in equal steps of 50, 100, 150, etc. kilogram and the corresponding readings are noted from the vertical scale of the microscope. The same procedure is repeated during unloading. The readings are noted from the vertical scale of the microscope. The readings are tabulated and the mean elevation (y) for a particular mass (M) can be calculated. The distance between the two knife edges (*l*) is noted and the distance between the point of suspension of the weight and the nearest knife edge (a) is also measured. The breadth (b) and the thickness (d) of the scale are determined by using vernier calipers and screw gauge respectively.

S.No	**Load (gm)**	**Microscope Readings (m)**		**Average**	**Elevation for a mass of m=50gm**
		Loading	**Unloading**		
1	W			a	a ≈ b
2	W+50			b	b ≈ c
3	W+100			c	c ≈ d
4	W+150			d	d ≈ e
5	W+200			e	e ≈ f
6	W+250			f	f ≈ g
7	W+300			g	
				Average y	

Young's modulus $Y = \frac{3Mgal^2}{2bd^3 y}$ Nm^{-2}

By substituting all the values in the above equation we can calculate the Young's modulus of the given metre scale.

1.13.2 THEORY OF NON UNIFORM BENDING

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Let us consider a beam of length l is supported on the two knife edges A and B as shown in Fig.1.19. The load of Mg is suspended using hanger at the centre point C of the beam. It is observed that the beam bends and the maximum displacement from the exact mid-point of the beam. Due to the load Mg is applied, the middle of beam react Mg/2 is acted vertically upwards at each knife edges. The bending is called as non-uniform bending. The beam may be treated as two cantilevers, whose free end carries a load Mg/2 respective each of length l/2.

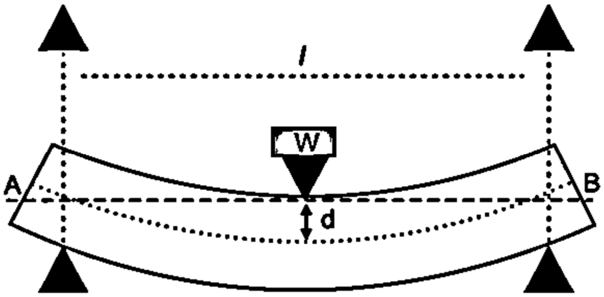

Fig. 1.19 Non-uniform bending Method

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Hence the elevation of A above D similar that the depression of D below A. we know that the depression of cantilever y = $\frac{Mgl^3}{3YI_g}$

Substituting the value of l and Mg in the above expression to find depression of a cantilever

$$y = \frac{(Mg/2)(l/2)^3}{3YI_g}$$

$$y = \frac{Mgl^3}{48YI_g}$$

But Ig = $\frac{bd^3}{12}$ for rectangular bar. Here b is the breadth and d is the thickness of the bar.

$$y = \frac{Mgl^3}{48Y} \times \frac{12}{bd^3}$$

$$y = \frac{Mgl^3}{4bd^3Y}$$

1.13.3 EXPERIMENTAL DETERMINATION OF YOUNG'S MODULUS BY NON UNIFORM BENDING METHOD.

The given beam is made to rest over two knife-edges. The length l of the beam between the knife-edges is noted. A weight hanger is suspended at the centre of the beam O where a pin is fixed vertically. A travelling microscope T is adjusted in front of this arrangement so that the tip of the pin is seen in the field of view of the microscope.

Initially, the beam is brought into elastic mood by loading and unloading it several times. Now, with the weight hanger alone as the load, the travelling microscope is adjusted until its horizontal cross-wire coincides with the tip of the pin. The corresponding vernier scale reading is noted.

Then weights are added (loading) to the weight hanger in steps of 50 gms upto to a total of 250 gms. In each case, it is ensured that the horizontal cross-wire coincides with the tip of the pin as in Fig 1.20 and the corresponding vertical scale readings are noted. The procedure is repeated by removing the weights (unloading) in the same steps of 50 gms each and the microscope readings are taken as before for the corresponding loads.

The mean of loading and unloading readings for each load M is recorded. The breadth b and the thickness t of the beam are measured by vernier caliper and screw gauge respectively. The observed values are substituted in the formula of Young's modulus to get the young's modulus of the material of the given beam by non-uniform bending.

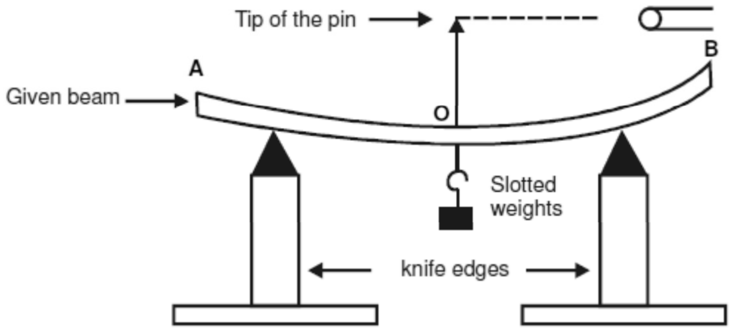

Fig. 1.20 Non-uniform bending experiment

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S.No	Load (gm)	Microscope Readings (m)		Average	Elevation for a mass of m=50gm
		Loading	Unloading		
1	W			A	a ≈ b
2	W+50			B	b≈c

3	W+100	c	$c \approx d$
4	W+150	d	$d \approx e$
5	W+200	e	$e \approx f$
6	W+250	f	$f \approx g$
7	W+300	g	
			Average y

$$\textbf{Young's modulus } Y = \frac{Mgl^3}{4bd^3y}\ \text{Nm}^{-2}$$

By substituting all the values in the above equation we can calculate the Young's modulus of the given metre scale.

1.14 I-SHAPED GIRDERS

In general, a girder bridge is very likely the most commonly built and utilized bridge in the world. Its basic design, in the most simplified form, can be compared to a log ranging from one side to the other across a river or creek

A girder supported at its two ends on the opposite walls of a room, bends under its own weight and, or under the load placed above it. The middle portion gets depressed. The depression (y) at the mid point of a rectangular beam is proportional to $\frac{Mgl^3}{Ybd^3}$. For the depression (y) to be small for a given load (w), the length of the girder (l) should be small and its breadth (b), depth (d) and Young's modulus for its material (Y) must be large.

Due to depression, the upper parts of the beam above neutral surface contract, while those below the neutral surface extended. Hence the stresses have a maximum value at the top and bottom and progressively decrease to zero as we approach the neutral surface from either face.

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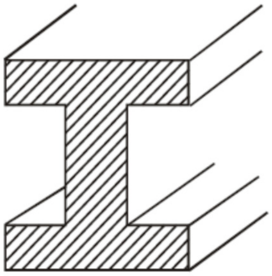

Fig.1.21 I shaped girders

Therefore, the upper and lower surfaces of the beam must be stronger than the intervening part. That is why the two surfaces of a girder or iron rails (f or railway tracks etc) are made much broader than the rest of it, thus giving its cross section the shape of the letter I. Hence the girders with I shape are called I-form girders. In this manner, material will be saved, without appreciably impairing its strength. Since the stress experienced at the middle portions of the girders is maximum, removal of the material does not reduce the strength of the girders.

Hence the advantages of I-shaped girders are:

- Economy of material without loss of efficiency.
- Self-loading is minimized.
- Strength of the girders does not fall.

Applications of I shape girders

1. They are used in the construction of bridges over the rivers or roads.
2. They are very much useful in the production of iron rails which are used in railway tracks.
3. They are used in the construction of iron beams to support the bridges for the heavy vehicles.
4. They are used as supporting beams for ceiling in the construction of buildings.

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SOLVED PROBLEMS

1. **A load of 4.0 Kg is suspended from a ceiling through a steel wire of radius 2.0 mm. Find the tensile stress developed in the wire when equilibrium is achieved .Take $g=3.1\,\pi$ m/s^2**

Solution : The Tension of the wire is

$F = 4.0 \text{ X } 3.1\,\pi$ N.

The area of cross-section is

$A = \pi r^2 = \pi \times (2.0 \times 10^{-3}\text{ m})^2$

$A = 4.0\,\pi \times 10^{-6}\text{ m}^2$

thus the tensile stress developed

$F/A = 4 \text{ X } 3.1\pi / 4.0\pi \times 10 \quad \text{N/m}^2$

Stress = $3.1 \times 10^6\ \text{N/m}^2$

2. **A steel wire of length 2 m is stretched through 2 mm . The cross-sectional area of the wire is 4 mm^2 . Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel = 2.0×10^{11} N/m^{-2}**

Solution :

The strain in the wire $\Delta l/l = 2\text{ mm}/2\text{m} = 10^{-3}$

The stress in the wire = Y x Strain

$= 2 \times 10^{11}\ \text{N/m}^{-2} \times 10^{3} = 2.0 \times 10^{8}\ \text{N/m}^2$

The volume of the wire = $(4 \times 10^{-6}\text{ m}^2) \times (2\text{m})$

$= 8 \times 10^{-6}\text{ m}^3$

The electrical potential energy stored = ½ x stress x strain x volume

$= \frac{1}{2} \times 2 \times 10^{8}\ \text{N/m}^2 \times 10^{-3} \times 8.0 \times 10^{-6}\ \text{m}^3$

= 0.8 J

3. **A copper wire of 3 m length and 1 mm diameter is subjected to a tension of 5N. Calculate the elongation produced in the wire if the young's modulus of elasticity of copper is 120Gpa.**

Solution:

Young's Modulus $Y = FL/Al$

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l = FL/AY

$= (5 \times 3) / \pi \times (0.5 \times 10^{-3})^2 \times 120 \times 10^9$

$= 15.9 \times 10^{-3}$ m

4. **A torsion pendulum is made using a steel wire of radius 0.25mm and a sphere of diameter 3 cm. The rigidity modulus of steel is 80 GPa and density of the material of the sphere is 11300 kg/m^3. If the period of oscillation is 2 second, find the length of the wire?**

Given
$N = 80 \times 10^9$ pascals
r= 0.25 mm
Radius of sphere = 1.5 cm
Density $\rho = 11300\ kg/m^3$
T = 2 seconds

$$Moment\ of\ Inertia\ of\ the\ sphere\ I = \frac{2}{5}MR^2$$

$$Density\ \rho = \frac{Mass}{Volume}$$

$$Mass = \frac{4}{3}\pi R^3 \times \rho$$

$$I = \frac{2}{5} \times \frac{4}{3}\pi R^3 \times R^2\ \rho$$

$$Rigidity\ Modulus\ n = \frac{8\pi I l}{T^2 r^4}$$

$$n = \frac{64\pi^2 l R^5 \rho}{T^2 r^4 15}$$

$$l = 3.463\ m$$

5. **A cylindrical wire of length 1m and radius 5 mm is rigidly clamped at one end.Calculate the couple required to twist the free and through an angle 45^0. The rigidity modulus of the material of the wire is 200×10^9 Pa**

Given L = 1 m , r = 5 mm , $\theta = 45^0 = \frac{45^0 \times \pi}{360^0}$, n= 200×10^9 Pa

$$c = \frac{\pi\, n\, \theta r^4}{2L}$$

$$C = 154.056\ Nm$$

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6. A cantilever of steel fixed horizontally is subjected to a load of 225 gm at its free end. The geometric moment of inertia of the cantilever is 4.5×10^{-11} m^{4}. If the length of cartilever and Young's modulus of steel are 1 m and 200×10^{9} pa respectively.

Solution:
M=225g = 225×10^{-3}kg; Ig = 4.5×10^{-11}m^{4}; l = 1m ; g =9.8ms-2; Y= 200×10^{9} Pa.
Y = ?

$$y = \frac{Mgl^3}{3YIg}$$

$$y = \frac{225 \times 10^{-3} \times 9.8 \times 1^3}{3 \times 200 \times 10^9 \times 4.5 \times 10^{-11}}$$

$$\boldsymbol{y = 0.082m}$$

7. A uniform rectangular bar 1 m long, 0.02 m broad and 0.003 m thick is supported on its flat surface symmetrically on two knife edges 0.7 m a part, when the loads of 0.2 kg are hung from the ends, the elevation of the bar above its normal position is found to be 0.0022 m. Find the Young's modulus of the material of the bar. (a = 0.15 m).

Solution:
M= 0.2kg; g = 9.8ms^{-2}; l = 0.7m; b = 0.02m; d = 0.003m; y = 0.0022m; a = 0.15m.
Y = ?

$$Y = \frac{3Mgal^2}{2bd^3y}$$

$$Y = \frac{3 \times 0.2 \times 9.8 \times 0.15 \times 0.7^2}{2 \times 0.02 \times 0.003^3 \times 0.0022}$$

$$\boldsymbol{Y = 1.819 \times 10^{11} Nm^{-2}}$$

EXERCISE PROBLEMS

1. One end of a wire 2m long and 0.2 cm^{2} in cross section is fixed in a ceiling and a load of 4.8Kg is attached to the free end. Find the extension of the wire. Young's modulus of steel = 2.0 x 10^{11} N/m^{2} (g= 10 m/s^{2}) **[Ans : 2.4 x 10^{-6} m]**
2. One end of nylon rope of length 4.5 m and diameter 6 mm is fixed to a free limb. A monkey weighing 100N jumps to catch the free and stay there. Find the elongation of the rope and the corresponding change in the diameter. Young modulus of nylon = 4.8

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10^{11} Nm^{-2} and Poisson's ration of nylon. **[Ans : 8.8 x 10^{-9}m]**

3. Calculate the density of load under a pressure 2 x 10^{8} N/m^{2}. Density of lead = 11.4 x 10^{3} kg/m^{3} Bulk modulus of Elasticity = 8 x 10^{9} Nm^{-2} **[Ans : 11.685 x 10^{3} Kgm^{-3}]**
4. Evaluate the rigidity modulus of the wire of length 1m and diameter 10^{-2}m is fixed at one end and twisted at the other end through an angle of 70^{0} by applying a couple of value 0.01Nm. **[Ans: 8.281 X 10^{10} Nm^{-2}]**
5. A circular disc of mass 0.25 kg and radius 2.5 X 10^{-2}m is fixed perpendicularly to the end of a wire of length 0.3m and diameter 10^{-3}m. Find the rigidity modulus of the wire if the period of the oscillation is 1.05 s **[Ans: 3.418 X 10^{10} Nm^{-2}]**
6. Determine the young's modulus of the material of a rod, if it is bent uniformly over two knife edges separated by a distance 0.6 m and loads of 2.5 kg are hung at 0.18 m away from the knife edges. The breadth and thickness of the rod are 0.025 m and 0.005 m respectively. The elevation at the middle of the rod is 0.007 m. **[Ans: 1.088×10^{11}Nm^{-2}]**
7. A circular cantilever of radius 1.2 cm and length 1.5m is fixed at one end. In the other end, a load of 2kg is applied. The Young's modulus of the cantilever is 19.5×1010 Nm-2. Find the depression produced. **[Ans: 6.946×10^{-3}m]**

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QUESTION BANK – PROPERTIES OF MATTER

2 MARKS QUESTIONS

1. State Hooke's law. (L-1)
2. Define stress and strain. Write their units. (L-1)
3. Name the types of strains and define it. (L-1)
4. Distinguish ductile and brittle substance. (L-2)
5. Why bridges are declared unsafe after long use? Give reasons. (L-2)
6. What do you infer from stress-strain diagram? (L-2)
7. How does verify Hooke's law? (L-3)
8. Define hysteresis in elasticity? (L-1)
9. Which is more elastic, a steel wire or rubber? Justify. (L-5)
10. Define Poisson's ratio. (L-1)
11. Define lateral strain. (L-1)
12. Name the three types of elastic moduli? (L-1)
13. Define Young's modulus of elasticity and mention its unit. (L-1)
14. Define rigidity modulus of elasticity and mention its unit. (L-1)
15. Define bulk modulus of elasticity and mention its unit. (L-1)
16. Point out the factors affecting the elasticity of a material. (L-4)
17. Interpret temperature affect the elasticity? (L-2)
18. Interpret the addition of impurity atoms affect the elasticity of a material? (L-2)
19. Interpret annealing affect the elasticity? (L-2)
20. Define bending moment of a beam. (L-1)
21. Define neutral axis. (L-1)
22. Define uniform bending and why is it said to be uniform? (L-1)
23. Distinguish uniform and non-uniform bending. (L-2)
24. Point out the advantages of I shape girder? (L-2)
25. Define cantilever? (L-1)
26. When a wire is bent back and forth, it becomes hot. Justify? (L-5)
27. Define elastic fatigue. (L-1)
28. Define yield point. (L-1)
29. Define moment of a force? (L-1)
30. List out the uses of torsional pendulum? (L-1)
31. Define angle of twist? (L-1)
32. Draw stress-strain diagram for soft iron and ceramics. (L-1)
33. Define torsional pendulum.(L-1)
34. Define tensile strength and safety factor. (L-1)

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PART-B Questions

1. State and explain Hooke's law of elasticity. Draw stress-strain diagram and discuss the behavior of ductile material under loading. (L-2)
2. What are the factors affecting elasticity of material and explain it. (L-2)
3. Derive expression for the torsional couple per unit angular twist when a cylinder is twisted. (L-6)
4. Derive an expression for the period of oscillation of a torsion pendulum. How can it be used to determine the torsional rigidity of a wire? (L-6)
5. Describe the theory of torsion pendulum and explain a method to determine the moment of inertia of an irregular body. (L-3)
6. Derive an expression for the internal bending moment of a beam in terms of radius of curvature.
   (L-6)
7. What is cantilever? Obtain an expression for the depression at the loaded end of a cantilever whose other end is fixed assuming that its own weight is not effective in bending. (L-2)
8. Derive an expression for depression at the free end of a cantilever due to load. Describe an experiment to determine the Young's modulus of the cantilever material using this expression. (L-6)
9. A beam of length l, breadth b and thickness d is fixed at one end, the length and breadth being horizontal, and a load W is applied at the other end. If E is the Young's modulus of the material of the beam, derive an expression for the depression at the loaded end. (The weight of the beam may be assumed to be not affecting in bending) (L-6)
10. Describe with necessary theory, the method to determine the Young's modulus of the material of a rectangular bar by uniform bending. (L-5)
11. Explain with theory, the determination of Young's modulus of elasticity of the material of a beam, supported at its ends and loaded in the middle. (L-5)
12. Discuss I shape girders. (L-2)

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UNIT -2

QUANTUM PHYSICS AND SUPER CONDUCTIVITY

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UNIT -2 QUANTUM PHYSICS AND SUPER CONDUCTIVITY

THE WAVE FUNCTION

Waves on a string are described by the displacement $y\ (x,t)$ of the string. In the case of a sound wave in air, the pressure $p\ (x,\ t)$ varies in space and time. In electromagnetic waves, the fields E and B, are the ones that vary in space and time. Therefore, to characterize the de Broglie wave associated with a material particle, we require a quantity that varies in space and time. The variable quantity is called the **wave function** for the particle and is usually designated by Ψ which is a function of the coordinates (x,y,z) and time t.

The displacement of a wave can either be positive or negative. In analogy, the wave function $\Psi\ (x,y,z,t)$ can have positive as well as negative values. The uncertainty principle tells us that we can only get the probability of finding the particle at (x,y,z) at time t. As probability cannot be negative, $\Psi\ (x,y,z,t)$ cannot be a direct measure of the presence of the particle. Hence, Ψ as such is not observable. But it must in some way indicate the presence of the particle, as it represents the wave associated with the particle in motion. For a particle having a well-defined momentum, the uncertainty in co-ordinate is infinite. Hence, the wave associated with it will be of infinite extent. Then, a free particle moving along x-axis with a definite momentum is described by the plane wave

$$\Psi(x,t) = Ae^{i(kx-\omega t)} \qquad \text{(i)}$$

Where $k = 2\pi/\lambda$ is the propagation constant and ω is the angular frequency.

In three dimensions, $$\Psi(r,t) = Ae^{i(kr-\omega t)} \qquad \text{(ii)}$$

Such particles are non-localized particles. For a localized particle, the wave function would be represented by the wave packet.

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Physical significance of wave function

Wave function is the variable quantity that is associated with a moving particle at any position (x,y,z) and at any time 't', also it relates the probability of finding the particle at that point and at that time.

- The wave function Ψ is a variable quantity.

- It is associated with a moving particle i.e., $\Psi\ (x, y, z, t)$ where x,y,z is the position of the particle at any time t.
- It gives information about the behaviour of the particle.
- Also it gives the statistical relation between the particle and the wave.
- Ψ is complex quantity and also Ψ does not have any meaning.
- But $|\Psi|^2$ has physical meaning, where $|\Psi|^2 = \Psi^*\ \Psi$, $\Psi^*$is the complex conjugate of Ψ.
- $|\Psi|^2$Exemplifies the probability of finding the particle per unit volume.
- $\iiint |\Psi|^2\ d\tau$ represents the probability of finding the particle in a given volume $d\tau$ where $d\tau = dx.\ dy.\ dz$
- The probability has any value between 0 and 1.
- If $\iiint \Psi^*\ \Psi\, d\tau = 0$ there is no particle in the given range.
- If $\iiint \Psi^*\ \Psi\, d\tau = 1$ there is particle certainly in the given range.
- If $\iiint \Psi^*\ \Psi\, d\tau = 0.5$,then, there is 50% chance for finding the particle and 50% chance for not finding the particle.
- A wave function Ψ satisfying the above relation is called a normalized wave function.

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SCHRODINGER WAVE EQUATION

Classical theories of particles and waves are based on equations of motion. In quantum mechanics the state of a particle is described by the wave function Ψ (r,t). To study the motion of systems in a general way, one requires an equation of motion, which was derived by Erwin Schrodinger in 1926. This equation, called the Schrodinger equation, governs the propagation of matter waves.

Experiments have proved beyond doubt that particles exhibit wavelike properties and the wavelength of matter wave is given by the following equation as,

$$\lambda = {}^{h}\!/_{p} \qquad \text{(i)}$$

The energy E of the particle is related to the frequency through the relation given below as follows,

$$E = h\nu = \hbar\omega \qquad \text{(ii)}$$

Free Particle Wave Equation in One Dimension – Time Dependent Schrodinger Equation

A free particle moving along *x*-axis with a definite momentum (a non-localized particle) is described by the infinite plane wave is given by,

$$\Psi(x,t) = \text{A} \exp\,[i\,(kx - \omega t)] \qquad \text{(iii)}$$

We know that, $k = 2\pi/\lambda = 2\pi p/h = p/\hbar$ and $\omega = E/\hbar$ (iv)

Substituting the above values in Eq. (iii), we get

$$\Psi(x,\, t) = \text{A} \exp\,[\,\tfrac{i}{\hbar}\,(px - Et)] \qquad \text{(v)}$$

Differentiating Equation (v), with respect to *x*, we get

$$\frac{\partial\Psi}{\partial \mathrm{x}} = \left(\frac{ip}{\hbar}\right) \text{A} \exp\left[\frac{i}{\hbar}\,(px - Et)\right] = \frac{ip}{\hbar}\Psi$$

Multiplying both sides by $-\,i\hbar$, we obtain

$$-\,i\hbar\frac{\partial\Psi}{\partial \mathrm{x}} = p\,\Psi \qquad \text{(vi)}$$

On differentiation the above equation, we get

$$\frac{\partial^2\Psi}{\partial \mathrm{x}^2} = \left(\frac{ip}{\hbar}\right)^2 \text{A} \exp\left[\frac{i}{\hbar}\,(px - Et)\right] = -\,\frac{p^2}{\hbar^2}\Psi$$

Multiplying both sides by $-\frac{\hbar^2}{2m}$, we get

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{p^2}{2m}\Psi \qquad \text{(vii)}$$

Differentiating Eq. (v) with respect to *t* gives

$$\frac{\partial\Psi}{\partial \mathrm{t}} = \left(-\frac{iE}{\hbar}\right)\text{A} \exp\left[\frac{i}{\hbar}\,(px - Et)\right] = -\,\frac{iE}{\hbar}\Psi$$

Multiplying both sides by $i\hbar$, we get

$$i\hbar\frac{\partial\Psi}{\partial \mathrm{t}} = E\Psi \qquad \text{(viii)}$$

The right hand side of Equations (vii) and (viii) are equal, since for a classical particle

$$E = \frac{p^2}{2m} \qquad \text{(ix)}$$

Hence, equating the left hand sides, we get

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$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \quad \text{(x)}$$

Equation (x) is the one dimensional time dependent Schrodinger equation for a free particle.In three dimensions, the equation becomes

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right) = -\frac{\hbar^2}{2m}\nabla^2\Psi \qquad \text{(xi)}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Energy and Momentum Operators

Equations (viii) and (vi) can be written in the form

$$\left(i\hbar\frac{\partial}{\partial t}\right)\Psi = E\Psi \text{ and } \left(-i\hbar\frac{\partial}{\partial x}\right)\Psi = p\Psi$$

This suggests that the energy *E* and the momentum *p* can be considered as differential operators.

Thus, we get

$$E \rightarrow i\hbar\frac{\partial}{\partial t} \text{and } p \rightarrow -i\hbar\frac{\partial}{\partial x} \qquad \text{(xii)}$$

In three dimensions, we have

$$E \rightarrow i\hbar\frac{\partial}{\partial t} \text{andp} = -i\hbar\,\nabla \qquad \text{(xiii)}$$

General Wave Equation

The energy *E* (also called Hamiltonian) of a particle in a potential *V* (*r*, *t*) is given by

$$E = \frac{p^2}{2m} + V\,(r,t) \qquad \text{(xiv)}$$

Schrodinger guessed that the operator for position co-ordinate is *r* and that for time is *t* itself. Hence, the operator for the potential is *V* (*r*, *t*). Substituting the operators for *E*, *p*, and *V* (*r*, *t*) in Eq. (xiv) and allowing the resulting operator equation to operate on the wave function Ψ (*r*, *t*), we get

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(r,t)\right]\Psi(r,\, t) \qquad \text{(xv)}$$

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Equation (xv) is the time-dependent Schrodinger equation for a particle moving in a potential *V* (*r, t*). The presence of *i* in the equation makes the solution of Ψ, in general, a complex quantity.

Time Independent Schrodinger Equation

In many cases of interest, the potential *V* is a function of the co-ordinates only. In such cases, we can separate the variables in Eq.(xv) and obtain two equations, one depending on the variable *t* and the other on the variable *r*. we can write

$$\Psi(r, t) = \Psi(r)\phi(t) \qquad \text{(xvi)}$$

Where $\Psi(r)$is a function of space co-ordinates and $\phi(t)$ is a function of time only. Substituting Eq. (xvi) in Eq.(xv), we get

$$\Psi(r).\, i\hbar\frac{d\phi(t)}{dt} = \phi(t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\Psi(r)$$

Dividing both sides by $\Psi\phi$, we get

$$i\hbar\frac{1}{\phi}\frac{d\phi}{dt} = \frac{1}{\Psi}\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\Psi(r) \qquad \text{(xvii)}$$

The left hand side of this equation is a function of *t* only, while the right side is a function of *r* only. This is possible only if both sides are equal to a constant, say *E*.Equating each side to *E*, we get

$$i\hbar\frac{d\phi(t)}{dt} = \mathrm{E}\phi(t) \qquad \text{(xviii)}$$

and

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\Psi(r) = \mathrm{E}\,\Psi(r) \qquad \text{(xix)}$$

Equation (xix) is the **time independent Schrodinger equation** or simply the **Schrodinger equation**. It will be shown that the constant *E* is the total energy of the system.

Applications of Schrodinger Wave Equation

Schrodinger wave equation has many applications. Among these, a simple problem of a particle in a potential well is discussed here.

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PARTICLE IN A ONE DIMENSIONAL WELL

Consider a particle of mass ‘m’ trapped in a one dimensional potential well of width ‘L’ as shown in fig. Since the particle is treated as a wave, the well sets the boundary conditions on the wave. According to uncertainty principle a particle confined to a finite space cannot have zero kinetic energy. Here the particle can move freely between 0 and L at constant speed and with constant kinetic energy along x direction.

The regions x<0 and x> L are forbidden. The particle cannot leave the well. The potential energy function that describes the particle in this case,

$$V(x) = \begin{cases} 0 & 0<x<L \\ \infty x \le 0, x \ge L \end{cases}$$

Schrodinger wave equation,

$$H\psi = E\psi$$

Where, [H=KE+PE] H is Hamiltonian operator along x- direction.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

Rearranging the equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0 \longrightarrow (1)$$

For the region I and III, $V(x) = \infty$

$$\psi_I(x) = \psi_{III}(x) = 0$$

In the region II, $V(x) = 0$

Substituting this in equation 1, we get

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \longrightarrow (2)$$

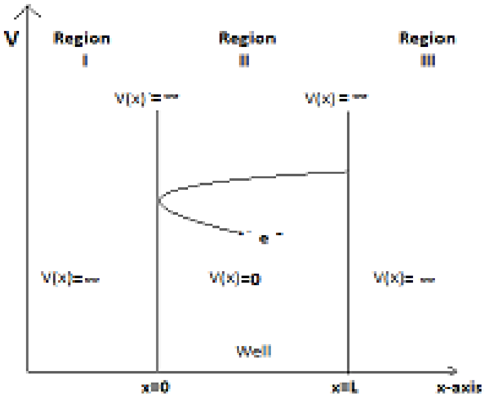

Fig. Partical in a one dimensional well

$$\frac{d^2\psi}{dx^2} + k^2 = 0 \longrightarrow (3)$$ where momentum vector $k=\sqrt{\frac{2mE}{\hbar^2}}$

Solution for this second order differential equation is,

$$\psi_{II}(x) = C\sin kx + D\cos kx \longrightarrow (4)$$

Where C and D are unknown constants to be determined. Since the particle cannot be found on the wall, and wave function is continuous, which means that $\psi(x)=0$ at both x=0 and x=L

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Use boundary conditions, to find constants C and D,

If x=0, then $\psi(0)=0$

Equation (4) becomes

$$C\sin 0 + D\cos 0 = 0$$

$$D = 0$$

If x=L, then $\psi(L)=0$

$$C\sin kL = 0$$

From the above equation, either C= 0 or sinkL=0

Already D=0 if C=0 then the solution is meaningless. Hence sinkL=0

Where kL= $0, \pm\pi, \pm 2\pi, \pm 3\pi \ldots. = n\pi$

Therefore $$k = \frac{n\pi}{L} \longrightarrow (5)$$

n=1, 2, 3….where n is any integer. Clearly n = 0 is not valid as the wavefunction vanishes.

Substituting C, D and k in equation (4) we get,

$$\psi_{II}(x) = C\sin\frac{n\pi x}{L} \longrightarrow (6)$$

Energy Eigen Value and Eigen Function

We know that, $k^2 = \frac{2mE}{\hbar^2}$

From equation (5), $k^2 = \frac{n^2\pi^2}{L^2}$

Equating these two equations, we get $E_n = \frac{n^2h^2}{8mL^2} \longrightarrow$ (7) (Generally E = E_n and $\hbar = \frac{h}{2\pi}$)

Equation (7) give the value of energy of a particle inside an infinitely deep potential well. It is clear that the energy of the particle is quantized. The discrete set of allowed energies are given by,

If n=1 $E_1 = \frac{h^2}{8mL^2}$

If n=2 $E_2 = \frac{4h^2}{8mL^2} = 4E_1$

If n=3 $\qquad E_3 = \frac{9h^2}{8mL^2} = 9E_1$

If n=4 $\qquad E_4 = \frac{16h^2}{8mL^2} = 16E_1$

Where E_n be the energy Eigen value and ψ_n be the Eigen function.

Zero point energy

Even at 0k, the lowest possible energy of a particle is not zero. that is If the energy is zero , there is no kinetic energy, and the particle is stationary. Then the linear momentum becomes zero ($E = \frac{P^2}{2m}$). And for the uncertainty in the particle's position to be infinite. (the particle can never be at rest because it always has some kinetic energy. If n=0,ψ_n becomes 0) This is called the zero point energy.

The unknown constant C may be found by normalizing the wave function (the particle is physically allowed to occupy somewhere in the space) , here the particle may not be found in regions of infinite potential, so the integration limits are 0 and L.

The total probability density of finding the particle in the well is 1

Hence the square modulus of the wave function is ,

$$\int_0^L |\psi_n(x)|^2 \, dx = 1 \qquad \longrightarrow \qquad (8)$$

$$\int_0^L C^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$C^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\frac{C^2}{2}\int_0^L \left[1 - \cos\left(\frac{2n\pi x}{L}\right)\right] dx = 1 \qquad \text{Where } \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\frac{C^2}{2}\left[x - \frac{\sin\left(\frac{2n\pi x}{L}\right)}{\frac{2n\pi}{L}}\right]_0^L = 1$$

$$C = \sqrt{\frac{2}{L}} \qquad \longrightarrow \qquad (9)$$

Substituting (9) in (6)

$$\psi_{II}(x) = \sqrt{\left(\frac{2}{L}\right)} \sin\frac{n\pi x}{L} \qquad \longrightarrow \qquad (10)$$

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Equation (10) gives the wave functions of a particle trapped in an one dimensional potential well of width L.

The Discrete energy levels , their corresponding normalized wave functions and probability densities are shown in fig. (a), (b) and (c)

Physical interpretation of probability density function

When n = 1 the probability of finding the particle inside the well between 0 and ¼ is 0.091 .It is maximum at x= ½ and zero at x = 0 , L. From this we can assume that at central region the probability of finding the particle is maximum and minimum at the walls.

When n = 2 the probability of finding the particle is maximum at x = L/4 and 3L/4 and zero at x = 0 ,L/2 and L ,means the particle is neither at the walls nor at the centre of the well.

When n= 3 ,ψ_3 is maximum values at x = L/6,L/2 and 5L/6 and zero at x = 0,L/3,2L/3 and L .means that the particle is neither at the walls nor at the centre of the well.

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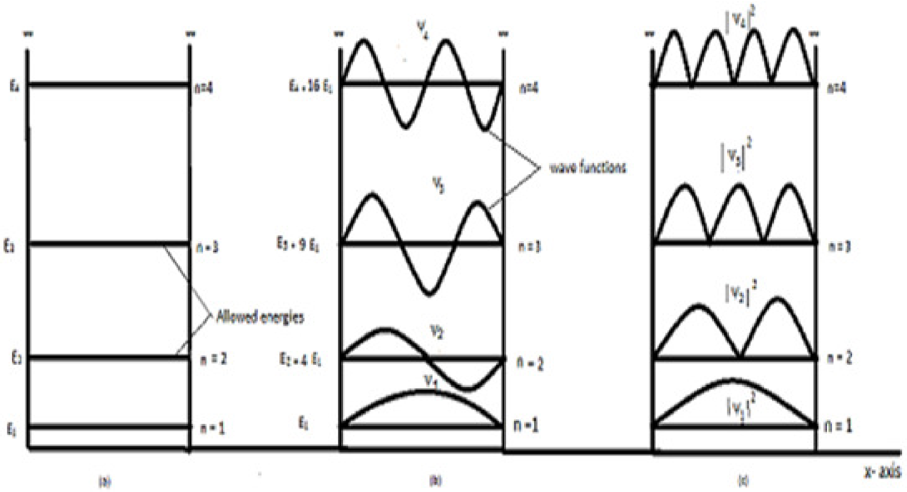

Fig. (a), (b) and (c) represents Discrete Energy levels ,wave function and probability density of a particle inside an infinite potential well

The Fermi-Dirac Distribution Function and Fermi Level

This distribution describes the occupation probability for a state of energy *E* at a temperature *T*. This is the ground state of the *N* electron system at absolute zero. When the temperature is increased, The kinetic energy of the electron gas increases with temperature. Therefore, some energy levels become occupied which were vacant at zero temperature, and some levels become vacant which were occupied at absolute zero. The distribution of electrons among the levels is usually described by the **distribution function, f(E) ,** which is defined as the probability that the level *E* is occupied by an electron. Thus if the level is certainly empty, then, $f(E) = 0$, while if it is certainly full, then $f(E) = 1$. In general, $f(E)$ has a value between zero and unity.

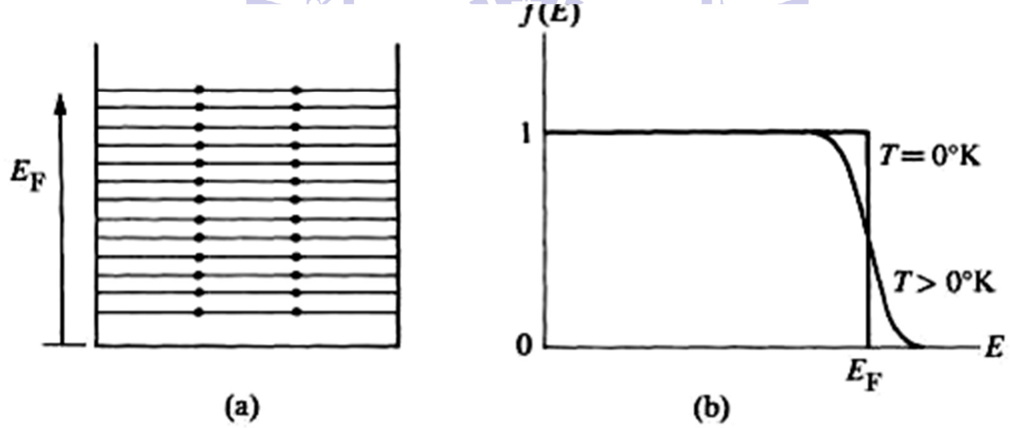

Fig. 1 (a) Occupation of energy levels according to the Pauli exclusion principle, (b) The distribution function $f(E)$, at $T = 0$°K and $T > 0$°K.

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It follows from the preceding discussion that the distribution function for electrons at $T = 0$°K has the form

$$f(E)=\begin{cases}1, & E<E_F\\0, & E>E_F\end{cases}.$$

That is, all levels below E_F are completely filled, and all those above E_F are completely empty. This function is plotted in Fig. 1(b), which shows the discontinuity at the Fermi energy.

When the system is heated (T>0°K), thermal energy excites the electrons. However, all the electrons do not share this energy equally, as would be the case in the classical treatment, because the electrons lying well below the Fermi level E_F cannot absorb energy. If they did so, they would move to a higher level, which would be already occupied, and hence the exclusion principle would be violated..

Therefore only those electrons close to the Fermi level can be excited, because the levels above E_F are empty, and hence when those electrons move to a higher level there is no violation of the exclusion principle. Thus only these electrons - which are a small fraction of the total number - are capable of being thermally excited.

The distribution function at non-zero temperature is given by the *Fermi distribution function*.

The conduction band in a piece of semiconductor consists of many available, allowed, empty energy levels. When calculating how many electrons will fill these levels and thus be counted in n, contributing to conductivity, we consider two factors:

- How many energy levels are there within a given range of energy, in our case the conduction band, and
- How likely is it that each level will be populated by an electron.

The likelihood in the second item is given by a probability function called the Fermi-Dirac distribution function. *f(E)* is the probability that a level (orbital) with energy E will be filled by an electron at thermal equilibrium, and the expression is:

$$f(E) = \frac{1}{1 + exp((E - E_F)/(k_B T))} \quad \text{or} \quad f(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1}.$$

Where: $\mathbf{k_B}$ is Boltzmann’s constant, 8.62×10^{-5} [eV/K],

T is the temperature in degrees Kelvin.

$\mathbf{E_F}$ is called the Fermi energy or Fermi level.

μ (the Greek letter mu) is the chemical potential.

This function is also plotted in Fig. 1(b), which shows that it is substantially the same as the distribution at T = 0°K, except very close to the Fermi level, where some of the electrons are excited from below E_F to above it.

The quantity $\boldsymbol{\mu}$ is called the chemical potential. The chemical potential can be determined in a way that the total number of electrons in the system is equal to *N*. At absolute zero $\boldsymbol{\mu} = \boldsymbol{E_F}$.

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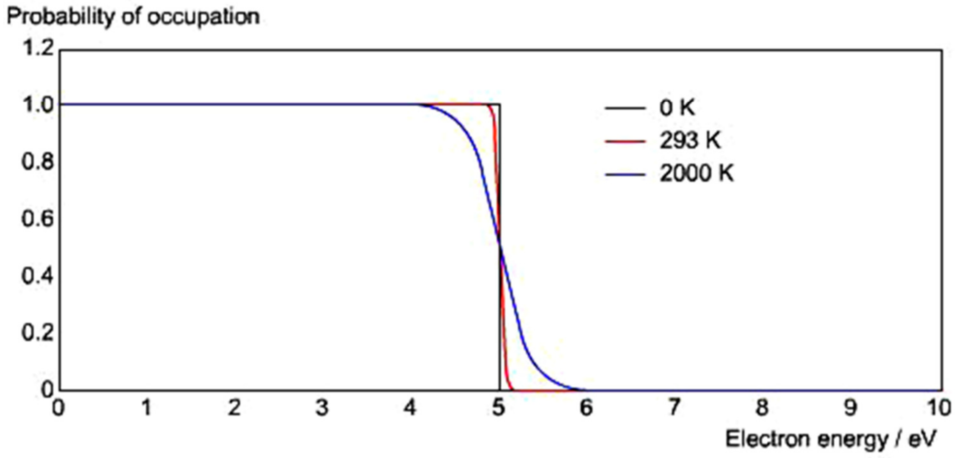
In order to understand the behavior of electrons at finite temperature qualitatively in metals and pure undoped semiconductors, it is clearly sufficient to treat μ as a constant to a first approximation. With this approximation, the Fermi-Dirac distribution can be plotted at several different temperatures. In the figure below, μ was set at 5 eV.



Fermi-Dirac distribution for several temperatures

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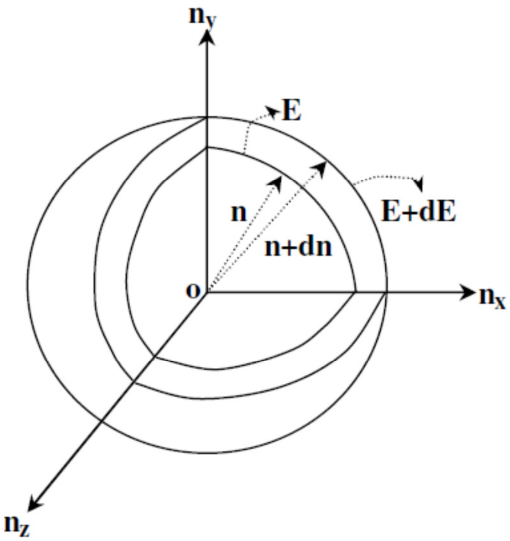
From this figure it is clear that at absolute zero the distribution is a step function. It has the value of 1 for energies below the Fermi energy, and a value of 0 for energies above. For finite temperatures the distribution gets smeared out, as some electrons begin to be thermally excited to energy levels above the chemical potential, μ. The figure shows that at room temperature the distribution function is still not very far from being a step function.

Density of states in metals and the carrier concentration in metals.

Density of state **Z(E) dE** is defined as the number of available energy states per unit volume in an energy interval **E** and **E + dE.**

Let us consider a cubical metal with sides ‘**a**’. The number of energy level can be obtained from various combinations of quantum numbers. A sphere is constructed with quantum numbers **n_x**, **n_y** and **n_z** in



three dimensional space **($n^2 = n_x^2 + n_y^2 + n_z^2$)**. In this space each point represents an energy level.

Let us construct a sphere of radius **n** in a space. The energy of this sphere is **E**. This sphere is divided into many spheres. Each sphere represents a particular combination of quantum numbers, particular energy value and particular radius. The energy of the next sphere is **E + dE** having radius **n + dn** from its origin '**O**'. The number of energy states within the sphere of energy **E** is equal to volume of the sphere.

The number of energy state within the sphere of radius $n = \frac{4}{3}\pi n^3$

Since the principle quantum number **n_x, n_y** and **n_z** can have only the positive integer value. The quantum numbers have positive value only lying in **1/8th** of the volume of the sphere. Therefore number of energy states within the sphere of energy E is equal to 1/8th of volume of the sphere

The number of energy state within the sphere of radius $n = \frac{1}{8}\left(\frac{4}{3}\pi n^3\right)$

Similarly, the number of energy states within the sphere of radius

$$n + dn = \frac{1}{8}\left(\frac{4}{3}\pi (n+dn)^3\right)$$

The number of available energy states between the shells of radius **n** and **n+dn** (or) between the energy levels E and E+dE = $n + dn = \frac{1}{8}\left(\frac{4}{3}\pi (n+dn)^3 - \frac{4}{3}\pi n^3\right)$

The number of available energy states between the energy interval dE is

$$Z(E)dE = \frac{1}{8}\left(\frac{4}{3}\pi (n^3 + dn^3 + 3n^2 dn + 3dn^2 n - n^3)\right)$$

Since the higher powers of dn^2 and dn^3 terms can be neglected $Z(E)dE = \frac{\pi}{6} 3n^2 dn$

$$Z(E)dE = \frac{\pi}{2} n^2 dn \qquad (1)$$

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From the application of Schrodinger wave equation, the energy of the electron in a cubical metal piece of sides **L** is given by $E = \frac{n^2h^2}{8mL^2}$

$$n^2 = \frac{8mL^2E}{h^2} \qquad (2) \qquad \text{(or)} \qquad n = \left(\frac{8mL^2E}{h^2}\right)^{1/2} \qquad (3)$$

Differentiating equation (2) we get $2ndn = \frac{8mL^2}{h^2}dE$

$$ndn = \frac{8mL^2}{2h^2}dE \qquad (4)$$

Eqn. (1) can be written as $Z(E)dE = \frac{\pi}{2}n(ndn)$

Substituting eqn. (3) and (4) in the above equation we have

$$Z(E)dE = \frac{\pi}{2}\left(\frac{8mL^2E}{h^2}\right)^{1/2}\left(\frac{8mL^2}{2h^2}dE\right)$$

$$= \frac{\pi}{2}.\frac{1}{2}\left(\frac{8mL^2}{h^2}\right)^{1/2}\left(\frac{8mL^2}{h^2}\right)E^{1/2}dE$$

$$= \frac{\pi}{4}\left(\frac{8mL^2}{h^2}\right)^{3/2}E^{1/2}dE$$

$$Z(E)dE = \frac{\pi}{4}\left(\frac{8m}{h^2}\right)^{3/2}L^3\ E^{1/2}dE$$

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If L^3=1, for unit volume, then we can write the number of available energy states per unit volume

Density of states $Z(E)dE = \frac{\pi}{4}\left(\frac{8m}{h^2}\right)^{3/2}E^{1/2}dE$

Since each energy level provides s electron states one with spin up and another with spin down

Density of energy states $Z(E)dE = 2\frac{\pi}{4}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}dE$

$$\boxed{Z(E)dE = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}dE}$$

Carrier concentration in metals:

The number of electrons available between the energy interval E and E+dE can be obtained by using density of state and Fermi function.

We know density of state $Z(E)dE = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}dE$

Fermi probability function $F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{K_B T}\right)}$

Where E_F=Fermi Level and K_B=Boltzmann constant

The carrier concentration that is the number of electrons per unit volume in a given energy is given by,

$$N(E)dE = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}dEF(E) \qquad (4)$$

Eqn. (4) represents the carrier concentration in metals.

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Fermi energy at 0 K

We know at 0 K maximum energy level that can be occupied by the electron is called Fermi energy level (E_{Fo})

0 K for E< E_F Therfore F(E)=1

Integrating Eqn. (4) with in the limit 0 to E_{Fo}

$$\int_0^N N(E)dE = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} \int_0^{E_{F0}} E^{1/2}dE$$

$$N = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} \frac{E_{F0}^{3/2}}{3/2}$$

$$E_{F0}^{3/2} = \frac{3N}{\pi}\left(\frac{h^2}{8m}\right)^{3/2}$$

$$\text{Fermi energy } E_{F0} = \left(\frac{3N}{\pi}\right)^{2/3} \frac{h^2}{8m}$$

ELECTRONS IN PERIODIC POTENTIAL

Electrons under the influence of a static, periodic potential V (x), i.e. such that it fulfils

V (x) = V (x + R), where R is a lattice vector.

Bloch's theorem states that the one-particle states in a periodic potential can be chosen so that

ψ (x) = u k (x) exp (i k · x) ,

where u k (x) is a periodic function with the periodicity of the lattice, and k belongs to the Brillouin zone. This implies that

ψ (x + R) = exp (i k · R) ψ (x)

i.e. ψ (x) is a periodic function up to a phase.

let us consider a function

u k (x) = exp (−ik · x) ψ (x)

After a translation we have

u k (x + R) = exp (−ik · x) exp (−ik · R) ψ (x + R)

= exp (−ik · x) ψ (x) = u k (x)

i.e. u k (x) is a periodic function with the periodicity of the lattice.

This means that ψ (x) = u k (x) exp (ik · x)

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ENERGY BANDS IN A SOLID

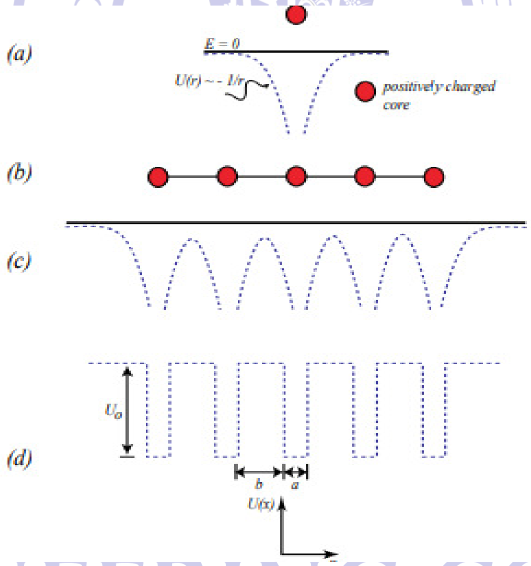
Electrons motion in a crystal will now be considered from a quantum mechanical point of view based on Schroedinger's equation.

The crystal may be viewed as a periodic arrangement of positively-charged ion cores comprising the positively charged nuclei and non-valence electrons that may be assumed stationary in the lattice points.

The potential energy of a valence electron in a single isolated atom is shown in Fig. a.

If several of these atoms are placed in close proximity and equidistant from each other (Fig. b), the potential functions of neighboring atoms overlap so that the net potential function will look like the potential function depicted in Fig. c.

However we can simplify the potential function by using the potential function shape shown in Fig. d (referred to as Kronig-Penny Model)



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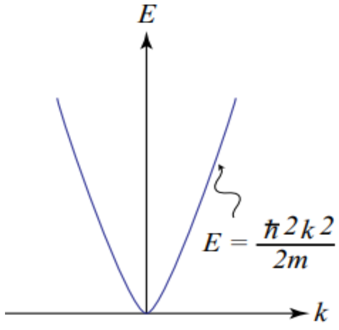
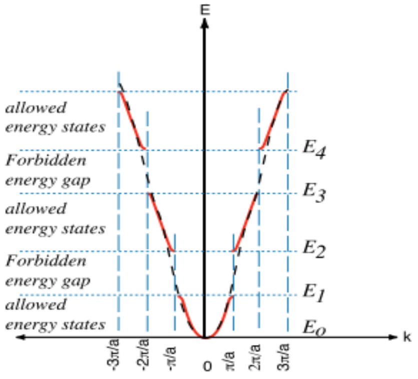
Relation between E and k





From the figure we can deduce the following information on the motion of an electron in a single crystal

There are only certain energy values that can be taken by the electron in a crystal.

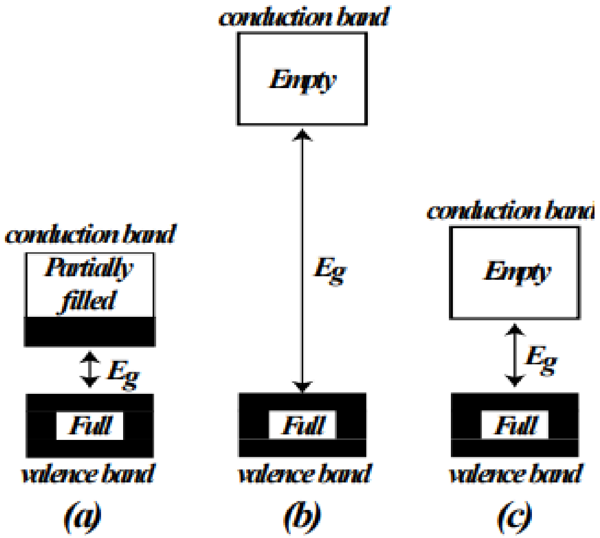
We may compare this to the E versus k plot for the free electron where the energy of the electron can take any value.

This energy range is called a band.

A band has a quasi (Latin for "almost")-continuous set of energy states which can be occupied by the electron.

The energy range between E1 and E2 is not accessible to electrons in the crystal. That is to say, the energy values between E1 and E2 are all forbidden.

This energy zone bounded by E1 and E2 is called the forbidden energy gap or simply energy gap.



(a) METALS **(b) INSULATORS** **(c) SEMICONDUCTORS**

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Electrical conduction will occur if the highest occupied energy band is partially filled (Fig. (a)). This is the case in metals where the highest energy band is partially filled and there are higher energy states in the band available for the electrons in the band to go to if their energies are somewhat increased by an externally applied electric field.

This highest energy band is called the conduction band. The electrons occupying this band are referred to as the conduction electrons. The reason they are called conduction electrons is that they are the electrons that are able to carry electric current across the solid.

An alternative name to these electrons is "free" electrons as these electrons are delocalized and are free to move throughout the solid if perturbed for instance by an applied electric field.

These electrons are not bound to any particular atom in the solid but instead they belong to the whole solid.

In insulators, however, all the energy bands are completely full with electrons (Fig. (b)). In this case the first empty band is separated by a forbidden energy gap from the highest filled band as we described in the energy band scheme of solids in the previous section.

The highest completely filled band in an insulator is called the valence band. It is called the valence band because it contains valence electrons ; i. e., the electrons in this valence band are somewhat bound to their individual atoms.

These electrons are , therefore, unable to move in the entire solid and they are consequently unable to carry electric current.

The separation in energy between the top of the valence band and the bottom of the next empty band (the conduction band) is the energy gap and is generally denoted by the symbol Eg .

In an insulator Eg is very large and it is unlikely that electrons in the valence band will be able to make it to the conduction band and, hence, the material remains insulating. * The band scheme of a semiconductor is essentially similar to that of the insulator ; the highest band is completely full with electrons (Fig. c)).

However in a semiconductor the energy gap Eg is small and electrons in the valence band can potentially acquire enough energy to surmount this energy gap and make it to the conduction band.

These electrons will, therefore, become free and available for conduction in the presence of an electric field across the semiconductor.

Generally, in a semiconductor this energy Eg required to raise the electron to the conduction band can be supplied as thermal or optical energy.

In silicon, the most known and utilized semiconductor, Eg ~ 1 eV and this amount of energy is comparable to the thermal energy of an electron at temperatures below liquid nitrogen temperature (~78K).

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Tunneling effect

Light can penetrates the air barrier because the wave has spread out nature. Same thing happens at the sub atomic level.

Tunneling was first observed by F. Hund in 1927.Tunneling is the quantum mechanical phenomenon. In which the particle can passes through the barrier, where energy of the tunneled particle is same but the probability amplitude is decreased. The barrier can be a region of high potential energy, a vaccum or an insulator.

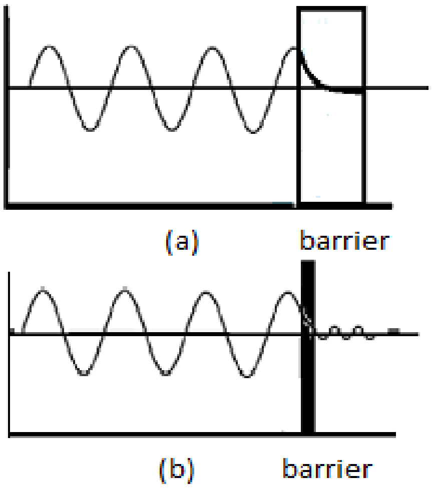

Fig. Tunneling of an electron through a potential barrier

In the quantum mechanics the electrons exhibit both wave and particle behavior. Tunneling is an effect of wave like nature. When particles (wave) encounters the barrier, a part of the wave pass through the barrier and therefore some of the particles may appear on the other side of the barrier without actually overcoming it or breaking it down. Tunneling is very much depend on the thickness of the barrier. If the barrier is thick, the wave decay quickly inside the barrier Fig.(a).Thinner the barrier more likely the wave can pass through it fig.(b). In the way ,if an electron passes through the barrier is called Tunneling.

Scanning Tunneling Microscope

The Scanning Tunneling Microscope was invented in 1981 at the IBM research laboratory by Gerd Binning and Heinrich Rohrer who shared Nobel Prize for Physics in 1986.

The STM is an electron microscope that works with single atom tip to get atomic resolution.

It is non optical device and uses the interactions of subatomic particles to produce images.

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Principle

The basic principle of STM is Quantum Tunneling. When two conductors brought close to each other by 10Å or more a potential difference is created between them, the electrons tunnel through the potential barrier (air) between these two conductors.

STM needs extremely clean, stable and vibration less surfaces and sharp tips.

Instrumentation

The main components of STM are

1.Scanning Tip 2. Piezoelectric scanner 3. Distance control and scanning unit 4.Tunneling current Amplifier 5. Data processing and display unit

Scanning Tip:

STM tips are usually made up of metal with high conductance like Tungten metal or platinum – iridium alloy and at the very end of the tip (called apex) there is one atom of the material. The tunneling current is carried by that particular atom.

Piezo electric scanner:

The tip is attached to the piezo electric tube scanner with three electrodes x,y and z. x and y electrodes are used to control the horizontal movement of the tip and the z is used to control the height of the tip. By adjusting the voltage on the piezo electric element , the distance between the tip and the surface of the specimen can be regulated.

Distance control and scanning unit:

A distance control is needed to position the tip on the sample before piezo electric control can take over for fine position and scanning unit is used to control how the tip move from one place to other place..

Tunneling current Amplifier:

The tunneling current generated is weak and to amplify the weak current, amplifier is connected to the tip and scanning unit.

Data processing and display unit:

The computer records the tunneling current and displays the 3 – D image of the sample surface.

Working:

In STM, electron is a source and behaves like a wave. First connect tip and the sample with particular voltage. The distance between the tip and the sample should be maintained in few Angstrom unit (4-7 Å) in order to get desired resolution. The voltage bias will cause electron tunneling so the electric current will start flow from tip to sample. The amount of current generated is indirectly proportional to the voltage. If low voltage is given, high current will flow through it and high voltage will produce low current.

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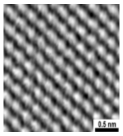

Fig. Graphite STM image

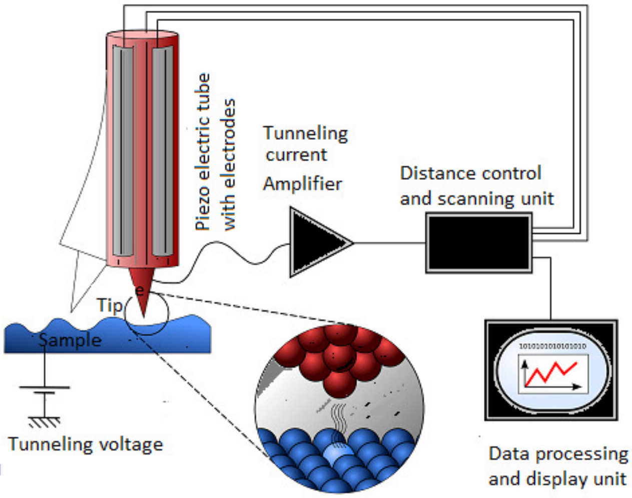

Fig. Schematic diagram of Scanning Tunneling Microscope

If the tip is moved across the surface of the sample in x-y plane, the changes in surface height cause changes in current. The change in current with respect to height of the tip corresponding to constant current is measured .There are two modes:

1. constant current mode
2. constant height mode

Constant current mode

The tip is allowed to move on the uneven surface of the sample. If the tip is close to sample the current flow will be more and If the distance between the tip and the sample increases the current flow will be less . In order to maintain the constant current we need to position the tip throughout the scanning period. Height can be adjusted by a voltage to the piezo electric height control mechanism to keep the current constant. If the tip goes up, the detector detects higher surface and If it goes down, it shows lower surface and so on.

Constant height mode

In this mode we have to maintain constant height and voltage . The tip is allowed to move on the uneven surface of the sample. The uneven surface brings change in current. Sometimes there will be a high current flow and sometimes low current will flow based on the distance between the tip and the surface of the sample . It is faster than constant current mode in which piezo electric movements require more time .

This mode is applicable for real flat surface (not more than 7 Å) and less preferred due to the risk of damaging the tip.

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The current is amplified .And this current is fed to a computer to generate an image of the atomic surface of the sample.

Advantages:

1. High resolution.(lateral 0.1nm and depth 0.01nm)
2. STM provides 3-D image.
3. It can be operated on variety of surfaces like air,water and vaccum and temperatures.
4. During electron tunneling the specimen is not demaged.

Disadvantages:

1. costly
2. Fragile
3. Needs trained persons to operate.

Applications:

1. The surface structure and the electronic property of the materials can be measured.

2. It is used to image the individual atom. And also used to rearrange the atom's position.

3. It is used in the field of Materials science, Metallurgy, Nano Science etc.

Introduction to Superconductivity

Superconductivity is a phenomenon of exactly zero electrical resistance and expulsion of magnetic flux fields occurring in certain materials, called superconductors, when cooled below a characteristic critical temperature. ***If mercury is cooled below 4.1 K, it loses all electric resistance. This discovery of superconductivity by H. Kammerlingh Onnes in 1911 was followed by the observation of other metals which exhibit zero resistivity below a certain critical temperature. The fact that the resistance is zero has been demonstrated by sustaining currents in superconducting lead rings for many years with no measurable reduction. An induced current in an ordinary metal ring would decay rapidly from the dissipation of ordinary resistance, but superconducting rings had exhibited a decay constant of over a billion years,***

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Properties of superconductors

1. Zero electrical resistance

The electrical resistance of the superconductor is zero below a transition temperature (T_c). it is the quick test to prove the superconductivity. This property of zero resistance is known as defining property of a superconductor.

2. Effect of magnetic field

The minimum magnetic field strength required to destroy the superconducting property at any temperature is known as critical magnetic field (H_c). the relation between critical magnetic field and temperature is given by

$$H_c = H_0[1 - \left(\frac{T}{T_C}\right)]$$

Where

H_c is the critical magnetic field

H_0 is critical magnetic field at absolute zero temperature of the material

3. Effect of electric current

The application of very high electrical current to a superconducting material destroys its superconducting property. The critical current (i_c) required to destroy the superconducting property is given by

$$i_c = 2лrH_c$$

Where H_c is the critical magnetic field

R is the radius of the superconducting wire

4. Persistent current

When a superconductor in the form of a ring is placed in a magnetic field, then the current is induced in it by electromagnetic induction. It flows indefinitely without any decrease in its value.

Such a steady current which flows through a superconducting ring without any decrease in its strength as long as the material is in superconducting state is called persistent current. The current persists even after the removal of the magnetic field.

5. Meissner effect

When a bulk sample of a conducting material is placed in a uniform magnetic field of flux density B, the magnetic lines of force penetrates through the material as shown in figure. However, when the material is cooled for the superconductivity below its transition temperature i.e, $T \leq T_c$, the magnetic flux originally present in the specimen is pushed out from the specimen as shown in the figure. Thus, inside the bulk superconducting specimen, magnetic induction (B) is zero. This phenomenon is known as Meissner effect.

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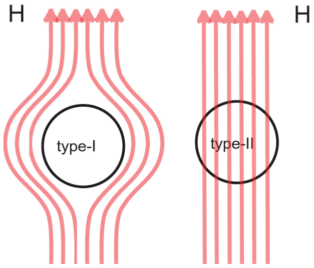

6. Isotope effect

The isotope of a superconducting element is also a superconductor. The presence of isotopes slightly changes the transition temperature of the superconductor. Generally, the transition temperature of a heavier isotope is lower than that of a lighter isotope.

Maxwell found that transition temperature is inversely proportional to the atomic masses of the isotopes of a single superconductor.

$$T_c \alpha \ 1/M^{\alpha}$$

$$T_c = \text{constant}/M^{\alpha}$$

$$M^{\alpha} \ T_c = \text{constant}$$

Where α is called isotope effect coefficient.

7. Josephson effect

The tunneling of superconducting electron pair (cooper pair) with opposite spin and momenta through a thin (10-20 Å) insulating barrier between two superconductors. This phenomenon is known as Josephson effect.

DC Josephson effect

The tunneling of superconducting electron pairs through Josephson junction leads to the flow of current without a voltage drop. This phenomenon is known as DC Josephson effect.

AC Josephson effect

When a DC voltage is applied across the Josephson junction through which supercurrent is flowing, an AC current (high frequency current oscillations) is noticed. This phenomenon is known as AC Josephson effect.

8. Effect of pressure

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By applying very high pressure, we can bring the transition temperature of a material nearer to room temperature, i.e, transition temperature is directly proportional to pressure at very high pressure.

9. Stress

If stress is applied to a material, then there is an increase in the dimension of the material, due to stress, an increase in transition temperature is noted and it also affects critical magnetic field.

10. Size

If the size of the specimen is reduced below 10^{-4} cm, the properties of superconducting state are modified.

Based on the magnetization behavior in an external magnetic field, the superconductors are classified into two types: They are

(I) Type – I superconductors

(II) Type – II superconductors

Type – I superconductors

In this superconductors, the magnetic field is totally excluded from the material below a certain magnetising field Hc. At Hc, the material loses its superconductivity abruptly and the magnetic field penetrates fully.

Characteristics of type I superconductors

1. They exhibit complete messiner effect i.e, they are completely diamagnetic.

2. They have only one critical magnetic field. The value of the critical magnetic field Hc at which the magnetization drop is very low. The maximum known crititcal magnetic field for type-I superconductor is of the order of 0.1 Tesla.

3. They are often called as soft superconductors because of their tendency to give away to low magnetic field.

4. The magnetization curve shows that transition at Hc is reversible.

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5. Below Hc, the materialnbehaves as a superconductor and above Hc it behaves as a normal conductor.

Example:

Most of the elemental superconductors are type I superconductors (except Niobium (Nb) and Vanadium (V)).

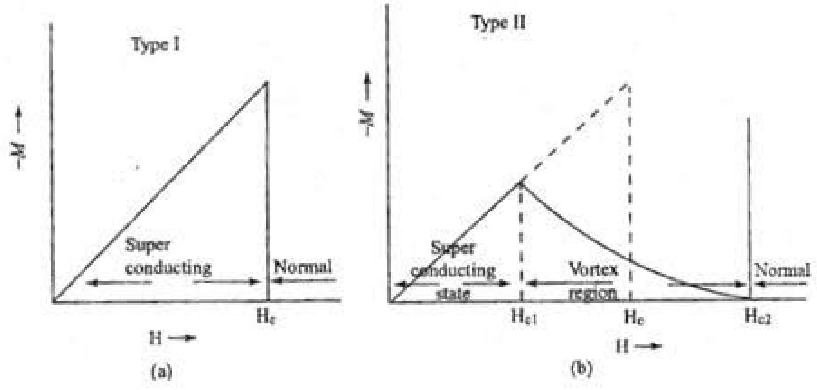
Disadvantages

(i) Type-I superconductors have only a limited practical applications because they cannot carry high currents

(ii) The low value of Hc makes these materials unsuitable for use in high-field superconducting magnets.

Type-II superconductor

Type-II superconductor is one in which the material loses its magnetization gradually rather than suddenly. They have two critical fields: Hc_1 (lower critical field) and Hc_2 (upper critical field). The specimen is perfect diamagnetic below Hc_1, i.e, magnetic field is completely excluded below Hc_1. At Hc_1, magnetic flux begins to penetrate the specimen and the penetration of flux continues until the upper critical field Hc_2 is reached. At Hc_2, magnetization vanishes and the specimen returns to normal conducting state.



Characteristics of type II superconductors

1. They do not show complete Meissner effect.

2. They are called as hard superconductors because they require a large amount of magnetic field to destroy superconducting state.

3. They do not behave as perfect diamagnetic materials above Hc_1.

4. Magnetization curve is reversible.

Examples:

Nb-Zr, Nb-Ti alloy and Nb-Sn intermetallic compounds

Applications of type-II superconductors

(i) They are used for power transmission without any loss.

(ii) They are technically more useful than type-I superconductors due to toleration of high magnetic fields.

BCS THEORY OF SUPERCONDUCTIVITY (QUALITATIVE)

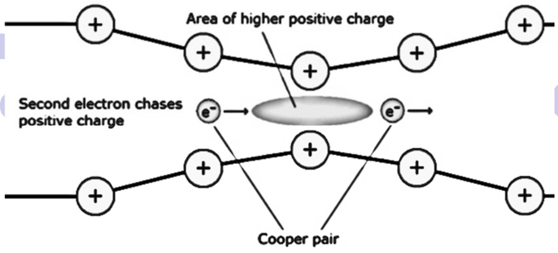
The microscopic theory of superconductivity developed by J. Bardeen, L.N. Cooper and J.R. Schrieffer in 1957, successfully explained the effects like zero resistivity, Meissner effect etc. This theory is known as BCS Theory.

Important feature of BCS theory

(i) Electrons from pairs (called cooper pairs) which propogate throughout the lattic.

(ii) The propogation of cooper pairs without resistance because the electrons move in resonance with phonons.

Therefore, the interaction described by the BCS theory is known as the electron – phonon interaction.



In a single lattice row, each electron experiences an attraction towards its nearest positive ion. When the electrons get very close to each other in the region between ions, they repel each other due to their mutual coulomb force.

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In an equilibrium condition, a balance between attraction and repulsion is established and the two electrons to from cooper pair. The collection of such pair (bosons) in a bulk sample condenses to form superconducting state. The electrons of the cooper pair propagating through the lattice as shown in the fig. The coulomb attraction between the electron and ions deforms the lattice which is propagated along with the electron.

This propagating wave is associated with phonon transmission and the electron-phonon resonance allows the electron along with its pair elsewhere in the lattice to move without resistance.

Predictions of BCS theory

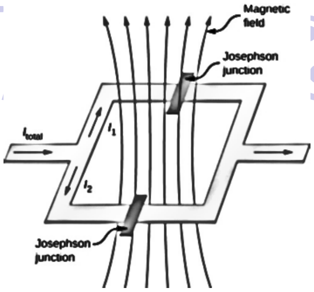
BCS theory successfully predicts the isotope effect, the variation of critical magnetic field with temperature and the existence of an energy gap between the ground state (superconducting state) and first excited state. It also predicts the qnantization of the magnetic flux through a superconducting ring,

$\Phi_o = h/(2e) = 2.068\times10^{-15}$ Tm^2 is referred to as the quantum fluxoid. The quantization of magnetic flux has been confirmed experimentally.

Applications of Superconductors

SQUID

It stands for **S**uperconducting **QU**antum **I**nterfernce **D**evice. It is an ultra sensitive instrument used to measure very weak magnetic field of the order of 10^{-14} tesla. A small change in magnetic field produces variation in the flux quantum.



Description and working

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A SQUID consists of a superconducting ring which can have the magnetic fields of quantum values (1, 2, 3..) of flux placed in between two Josephson junctions as shown in fig.. When the magnetic field is applied perpendicular to the plane of the ring, the current is induced at the two Josephson junctions. The induced current produces the interference pattern and it flows around the ring so that the magnetic flux in the ring can have the quantum value of magnetic field applied.

Application

(i)It is used to detect the variation of very minute magnetic signals in terms of quantum flux.

(ii)It can also be used as storage device for magnetic flux.

Magnetic Levitated Train (Maglev train):

When a magnet is placed over a superconductor, it floats. This phenomenon is known as magnetic levitation. This principle is used in magnetic levitation. The magnetic levitated train does not move over the rails but it floats above the rails.

Description and working

This train has superconducting magnets built into its base. An aluminium guideway, above which the train will be set afloat by magnetic levitation. It is brought about by enormous repulsion between two highly powerful magnetic fields, one produced by the superconducting magnet inside the train and the other by the electric current in the aluminium guideway.

The current in the guideway produces the necessary magnetic field to levitate the train and helps in propelling the train forward. The train is provided with retractable wheels. The wheels serve almost the same purpose as those of an airplane. The train runs on the guideway just as the way the airplane takes off.

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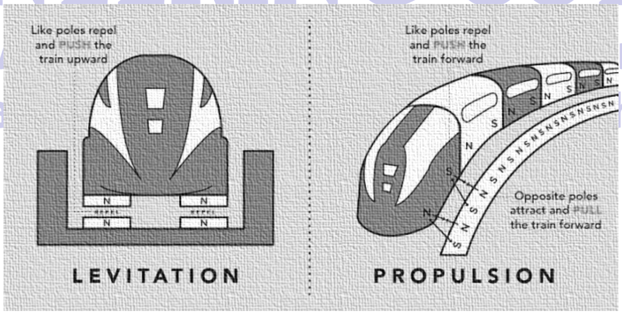

SOLVED PROBLEMS

1. An X-ray photon is found to have doubled its wavelength on being scattered by 90°. Find the energy and wavelength of incident photon

$$d\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

For θ = 90° $\cos 90 = 0$

$$d\lambda = \lambda' - \lambda = \frac{h}{m_0 c} = 6.6 * \frac{10^{-34}}{9*10^{-31}*3*10^{8}} = 0.02445 * 10^{-10} m$$

If λ is the wavelength of incident photon, the wavelength of scattered photon = $\lambda + \Delta\lambda$

But, $\lambda + \Delta\lambda = 2\lambda$

$$\lambda = \Delta\lambda = 0.02425 * 10^{-10}$$

Energy of photon $h\nu = \frac{hc}{\lambda}$

$$= 6.625 * 10^{-34} * 3 * 10^{8}/0.02425 * 10^{-10}$$

$$= 19.89 * 10^{-16}/0.02425 \ J$$

$$= 19.89 * 10^{-16}/0.02425 * 1.6 * 10^{-19} eV$$

= 0.51 MeV

2. Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 36° C.

Given T = 36° C. = 36 + 273 = 309 K

The peak wavelength is related to the surface temperature of the body. By using Wien's displacement law

$$\lambda_m = 2.898 * 10^{-3}/\mathrm{T}$$

$$\lambda_m = 2.898 * \frac{10^{-3}}{309} K$$

$$\lambda_m = 9.37 \, \mu \, m$$

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3. In a Compton scattering, the incident photons have wavelength 0.6nm. Calculate the wavelength of scattered radiation, if they are viewed at an angle of 60° to the direction of incidence.

Data : $h = 6.625 * 10^{-34}\, Js,$ $m_0 = 9.11 * 10^{-31} kg,$

$c = 3 * 10^8 \frac{m}{s},$ $\lambda = 0.6\, nm = 0.6 * 10^{-9}\, m,\ \ \theta = 60\,°$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\lambda' = 0.6 * 10^{-9} + \frac{6.625*10^{-34}\,(1-\cos 60)}{9.11*10^{-31}*3*10^{8}}$$

$$\lambda' = 0.6 * 10^{-9} + 3.3125 * 10^{-34}/27.33 * 10^{-23}$$

$$\lambda' = 0.6 * 10^{-9} + 0.1212 * 10^{-11}$$

$$= 0.6 * 10^{-9} + 0.001212 * 10^{-9}$$

$$= 0.6012\, nm$$

4. Determine the wavelength of an electron accelerated from rest through a potential difference of 10 volts.

Data : V = 10 volts

The de-Broglie wavelength λ is given by

$$\lambda = \sqrt{\frac{150}{V}} \times 10^{-10}\, m$$

$$\lambda = \sqrt{\frac{150}{10}} \times 10^{-10}\, m = 3.87 \times 10^{-10}\, m$$

5. Compute the de-Broglie wavelength of an electron whose kinetic energy is 100 eV.

Data: Kinetic energy (E_k) = 100 eV

The kinetic energy (E_k) = $p^2/2m$

Therefore, $p = \sqrt{2mE_k}$

$p = \sqrt{2 \times (9.11 \times 10^{-31}\, kg) \times (100\, eV) \times (1.6 \times 10^{-19} J/eV)}$

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$p = \sqrt{2.915 \times 10^{-47}}$

$p = 5.399 \times 10^{-24}$ kg m/s

$$\lambda = {}^{h}/_{p} = \frac{6.626\times 10^{-34}\ Js}{5.399\times10^{-24}\ kg\ m/s} = 1.22 \times 10^{-10}\ m = 1.22\ nm$$

6. An electron and a proton have the same non-relativistic kinetic energy. Show that the proton has a shorter de-Broglie wavelength.

We know that Kinetic energy, $E_k = p^2/2m$

Then, p = $\sqrt{2mE_k}$

The de-Broglie wavelength, $\lambda = {}^{h}/_{p} = \frac{h}{\sqrt{2mE_k}}$

Since proton and electron have the same kinetic energy, we have

$$\lambda_e/\lambda_p = \sqrt{m_p/m_e}$$

Since, $m_p > m_e$, we have $\lambda_p < \lambda_e$.

7. Calculate the energy difference between the ground state and first excited state of an electron in a one dimensional rigid well of width 10^{-7}cm.

Solution

The energy of a particle of mass m in a one dimensional well of length L is given as,

$$E_n = \frac{n^2h^2}{8mL^2}$$

Where n = 1,2,3....

$$E_n = \frac{n^2(6.63 \times 10^{-34})^2}{8\times9.1\times10^{-31} \times(10^{-7})^2} = n^2 6.03 \times 10^{-24} J$$

$$= \frac{n^2 6.03\times10^{-24}}{1.6\times10^{-19}} eV$$

$$= n^2 3.7737\times10^{-5} eV$$

For ground state, n =1

$E_1 = 3.7737\times10^{-5}$eV

For first excited state, n = 2

$E_2 = 2^2\times3.7737\times10^{-5} = 15.09 \times 10^{-5}$eV

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Difference in energy between the first excited state and the ground state can be given as,

$$\Delta E = E_2 - E_1 = 15.09 \times 10^{-5} - 3.7737 \times 10^{-5}$$
$$= 11.37 \times 10^{-5} \text{ eV}$$

8. A particle confined to move along the X – direction has the wave function ψ = ax between x = 0 and x = 1.0, and ψ = 0 elsewhere. Find the probability that the particle can be found between x = 0.45 and x = 0.55.

Solution

The probability of finding the particle between x_1 and x_2 in the nth state can be given as

$$P = \int_{x_1}^{x_2} |\Psi_n|^2$$

Given that $\Psi_n = ax$ between x =0 and x = 1 , and $x_1 = 0.45$ and $x_2 = 0.55$

Now,
$$P = \int_{0.45}^{0.55} (ax)^2 \, dx$$
$$= a^2 \int_{0.45}^{0.55} x^2 \, dx$$
$$= \frac{a^2}{3} [x^3]_{0.45}^{0.55}$$
$$= \frac{a^2}{3} [(0.55)^3 - (0.45)^3]$$
$$= \frac{a^2}{3} [0.166375 - 0.09112]$$
$$= \frac{0.075255}{3} a^2$$
$$= 0.025085 \, a^2$$

9. Find the probabilities of finding a particle trapped in a well of width L in the region from 0 to 0.25L for the ground state.

Solution

If a particle is trapped in a box of width L , then the wave function can be given as,

$$\Psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Now, the probability of finding the particle between 0 and 0.25L is given by,

$$P = \int_0^{0.25L} \left(\sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}\right)^2$$
$$= \frac{2}{L} \int_0^{0.25L} \sin^2 \frac{n\pi x}{L} \, dx$$
$$= \frac{1}{L} \int_0^{0.25L} \left[1 - \cos \frac{2n\pi x}{L}\right] dx$$
$$= \frac{1}{L} \left[x - \frac{L}{2\pi n} \sin \frac{2n\pi x}{L}\right]_0^{0.25L}$$

For n = 1

$$P = \frac{1}{L} \left[(0.25L - \frac{L}{2\pi} \sin(0.57\pi) - 0\right]$$
$$= 0.25 - \frac{1}{2\pi} \sin 90°$$
$$= 0.25 - 2.777 \times 10^{-3}$$
$$= 0.2472$$

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10. X rays of wavelength 0.120nm are scattered from a very thin slice of carbon. What will be the wavelengths of X –rays scattered at (a) 0°(b) 90°(c) 180°

Solution

Compton shift is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\lambda' = \lambda + 0.024(1 - \cos\theta) \qquad \text{where } \frac{h}{m_0 c} = 0.024\text{Å}$$

(a) When $\theta = 0°$

$$\lambda' = 1.20 + .024(1-1) = 1.20\ \text{Å}$$

(b) When $\theta = 90°$

$$\lambda' = 1.20 + .024(1-0) = 1.224\text{Å}$$

(c) When $\theta = 180°$

$$\lambda' = 1.20 + .024(1-(-1)) = 1.248\text{Å}$$

SELF EVALUATION QUESTIONS

SHORT ANSWER QUESTIONS

1. Define blackbody and black body radiation.
2. State Wien's displacement law.
3. State Rayleigh – Jean's law.
4. State Stefan-Boltzmann's law.
5. Interpret ultraviolet catastrophe?
6. Describe the postulates of Planck's Quantum theory.
7. Write down Planck's radiation formula in terms of wavelength.
8. What is Compton Effect? (Incoherent scattering)
9. Compute Compton wavelength?
10. Calculate minimum and maximum values of Compton shift?
11. Differentiate Compton wavelength and Compton shift.
12. Photo electric and Compton effect both arise due to the action of photon on electron but the two effects are not the same. Justify.
13. Out of visible light and X-ray which can most easily show Compton effect. Support your answer?
14. State Heisenberg's uncertainty principle.
15. Describe wave function?
16. Point out some of the physical significance of the wave function.

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17. Illustrate “Eigen values” and “Eigen functions”?
18. Define Photon? Give its properties.
19. Express de- Broglie wave or matter waves?
20. Explain degenerate states and non-degenerate states.
21. Express is Schrödinger wave equation?
22. Illustrate Schrödinger time independent and dependent wave equations.
23. Outline the importance of normalising a wave function?
24. Free particles in one dimensional box can have positive energy values only. Justify.
25. Write the energy value of the particle in one dimensional box.
26. Illustrate free particle? List out the parameters characterise it?
27. State the basic principle of Electron Microscope.
28. Point out any four differences between Optical and Electron microscope.
29. Illustrate any four failures of Classical Mechanics.
30. State the limitations of Optical microscope.
31. Define quantum tunnelling?
32. An X-ray photon is found to have doubled its wavelength on being scattered by 90º. Calculate the energy and wavelength of incident photon.
33. Calculate the de-Broglie wave length of an electron in the first Bohr orbit of Hydrogen atom.
34. A proton and a deuteron have the same kinetic energy. Determine the longer wavelength.
35. Calculate the uncertainty in the momentum and velocity of an electron confined in a box of length 10 nm.
36. Compute the energy of the lowest three levels for an electron in square well of width 3 Å.
37. Calculate the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 36 ºC.

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LONG ANSWER QUESTIONS

1. Illustrate Black Body radiation .Derive the expression for Planck theory of black body radiation?
2. Define Compton Effect? Derive an expression for the Compton shift.
3. Derive an expression for de-Broglie wavelength for matter waves?
4. Illustrate de-Broglie hypothesis? Describe an experiment, which supports the existence of matter waves.
5. Derive an expression for Schrodinger time independent and dependent wave equation.

6. Explain particle in one dimensional box and also explain three dimensional effects.
7. Explain Electron microscope .With neat sketch explain the Construction and working of Electron microscope.
8. Draw and explain the Scanning Electron Microscope. Write the differences between Scanning electron microscope and Transmission electron microscope.
9. Explain (i) Transmission electron Microscope (ii) Scanning Electron Microscope.
10. Explain scanningtunneling microscope with neat diagram.

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PHYSICS FOR BIOSCIENCE

UNIT III MAGNETIC AND DIELECTRIC MATERIALS

Magnetism in materials

It arises from the magnetic moment or magnetic dipole of the magnetic materials. When an electron revolves around the positive nucleus, orbital magnetic moment arises. Similarly when the electron spins, spin magnetic moment arises. Similarly when the electron spins, spin magnetic moment arises. Materials which can be magnetised by an external magnetic field are called magnetic materials. When these materials are kept in external magnetic field, they will create a permanent magnetic moment in it.

Basic terms involved in magnetism

1. Magnetic flux (ϕ)

It is the total number of magnetic lines of force passing through a surface. Its unit is weber (Wb)

2. Magnetic flux density or magnetic induction (B)

Magnetic flux density at any point in a magnetic field passing normally through unit area of cross section (A) at that point. It is denoted by the symbol B andits unit is weber / metre^2 or tesla.

$$\mathbf{B} = \frac{\phi}{A} \text{ weber / metre}^2 \text{ or tesla.}$$

3. Magnetic field intensity (or) strength (H)

Magnetic field intensity at any point in a magnetic field is the force experienced by unit north pole placed at that point.

It is denoted by H and its unit is Newton per weber or ampere turns per metre (A/m).

4. Magnetic permeability (μ)

Magnetic permeability of a substance measure the degree to which the magnetic field can penetrate through the substance.

It is found that magnetic flux density (B) is directly proportional to the magnetic field strength (H)

$$B \;\alpha\; H$$

$$B = \mu H$$

Where **μ** is a constant of proportionality and it is known as permeability or absolute permeability of the medium.

$$\mu = \frac{B}{H}$$

Hence, the permeability of a substance is the ratio of the magnetic flux density (B) inside the substance to the magnetic field intensity (H).

Absolute permeability

Absolute permeability of a medium or material is defined as the product of permeability of free space (μ_0) and the relative permeability of the medium (μ_r)

$$\mu = \mu_0 \text{x } \mu_r$$

Relative permeability of medium (μ_r)

Relative permeability of a medium is defined as the ratio between absolute permeability of a medium to the permeability of a free space

$$\mu_r = \mu / \mu_0$$

5. Intensity of magnetization (I)

The term magnetization means the process of converting non-magnetic material on magnetic material. The intensity of magnetisation (I) is the measure of the magnetisation of a magnetized specimen. It is defined as the magnetic moment per unit volume.

$$I = \frac{M}{V} \text{ weber / metre}^2$$

6. Magnetic susceptibility (χ)

Magnetic susceptibility (χ) of a specimen is a measure of how easily a specimen can be magnetized in a magnetic field.

It is the ratio of intensity of magnetisation (I) induced in it to the magnetizing field (H).

$$\chi = \frac{I}{H}$$

7. Bohr Magnetron (μ_B)

The orbital magnetic moment and the spin magnetic moment of an electron in an atom can be expressed in terms of atomic unit of magnetic moment called as Bohr magnetron.

$$1 \text{ Bohr magneton} = \frac{e\hbar}{2m} = \mu_B = 9.27 \times 10^{-24} \text{ Am}^2$$

Relation between susceptibility (χ) and Relative permeability (μr)

When a current is supplied through a coil, magnetic field is developed. When a magnetic material is placed inside an external magnetic field, the magnetic flux density (B) arises due to applied magnetic field (H) and also due to the induced magnetization (I).

i.e., the total flux density,

$$B = \mu_0 (H+I) \qquad 1)$$

$$\text{We know that, } \mu = \frac{B}{H} \Rightarrow \mu H \qquad 2)$$

Equating eq. 1) and 2) we get,
$\mu H = \mu_0 (H+I)$
Since $\mu = \mu_0\mu_r$ we have

$$\mu_0\mu_r H = \mu_0 H \left(1+\frac{I}{H}\right)$$

But $\frac{I}{H} = \chi$

$$\therefore \ \mu_r = 1+\chi$$

TYPES OF MAGNETIC MATERIALS

The magnetic materials are classified according to the presence or absence of the permanent magnetic dipoles.

⇒ The materials without permanent magnetic moment

Example: Diamagnetic materials.

⇒ The materials with permanent magnetic moment.

Example Paramagnetic materials
Ferromagnetic materials
Anti-Ferromagnetic materials
Ferrimagnetic materials.

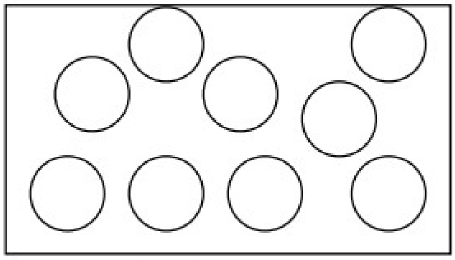
DIFFERENT TYPES OF MAGNETIC MATERIALS

DIAMAGNETIC MATERIALS

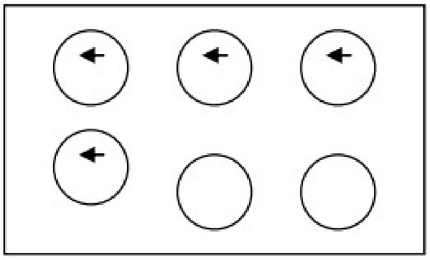
Diamagnetism is exhibited by all the materials. The atoms in the diamagnetic materials do not possess permanent magnetic moment. However, when a material is placed in a magnetic field, the electrons in the atomic orbits tend to counteract the external magnetic field and the atoms acquire an induced magnetic moment. As a result, the material becomes magnetized. The direction of the induced dipole moment is opposite to that of externally applied magnetic field. Due to this effect, the material gets very weakly repelled, in the magnetic field. This phenomenon is known as diamagnetism.

When a magnetic field Ho is applied in the direction shown in figure the atoms acquire an induced magnetic moment in the opposite direction to that of the field. The strength of the induced magnetic moment is proportional to the applied field and hence magnetization of the material varies directly with the strength of the magnetic field.

The induced dipoles and magnetization vanish as soon as the applied field is removed.



H = 0

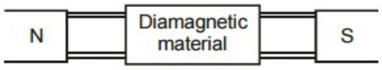


H = Ho

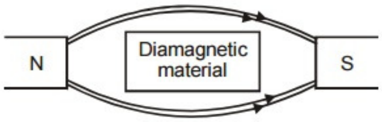
Properties of diamagnetic material

- They repel the magnetic lines of force, if placed in a magnetic field as shown in figure.
    - The susceptibility is negative and it is independent of temperature and applied field strength. ($\chi = -$ve).
    - The permeability is less than one.
    - There is no permanent dipole moment.
    - When the temperature is greater than the critical temperature diamagnetic becomes normal material.
    - It has superconducting property.

Examples : Gold, germanium, silicon, antimony, bismuth, silver, lead, copper, hydrogen, Water and alcohol.



(a) Normal State



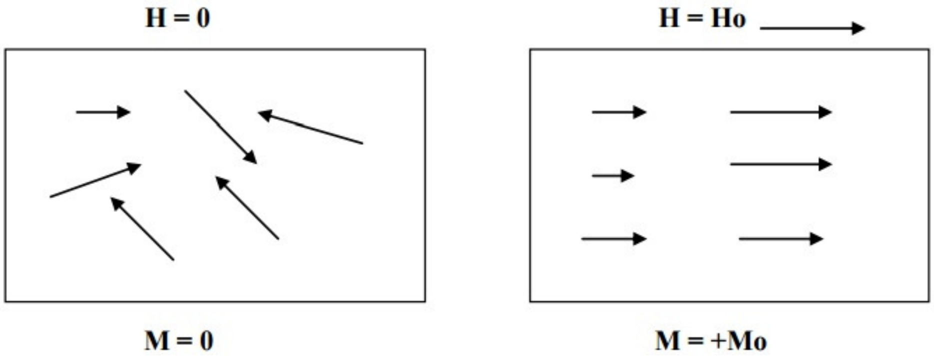
(b) Diamagnetic state

Fig. 1 Diamagnetic material

PARAMAGNETIC MATERIALS

In certain materials, each atom or molecule possesses a net permanent magnetic moment (due to orbital and spin magnetic moment) even in the absence of an external magnetic field. The magnetic moments are randomly oriented in the absence of external magnetic field. Therefore the net magnetic moment is zero, and hence the magnetization of the material is zero.

But, when an external magnetic field is applied, the magnetic dipoles tend to align themselves in the direction of the magnetic field and the material becomes magnetized as shown in fig. This effect is known as paramagnetism.



Thermal agitation disturbs the alignment of the magnetic moments. With an increase in temperature, the increase in thermal agitation tends to randomize the dipole direction thus leading to a decrease in magnetization. This indicates that the paramagnetic susceptibility decreases with increases in temperature. It is noted that the paramagnetic susceptibility varies inversely with temperature.

$$\chi \propto \frac{1}{T}$$

$$\chi = \frac{C}{T}$$

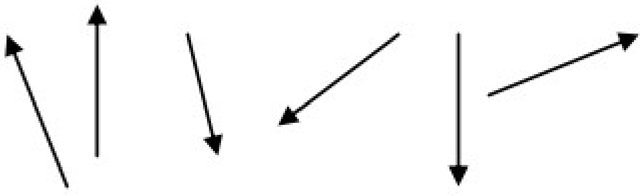
This is known as Curie law of paramagnetism and C is a constant called Curie constant

Properties of paramagnetic materials

- Paramagnetic materials attract magnetic lines of force.
- They possess permanent dipole moment.
- The susceptibility is positive and depend on temperature is given by

$$\chi = \frac{C}{T-\theta}$$

The spin alignment is shown in fig.



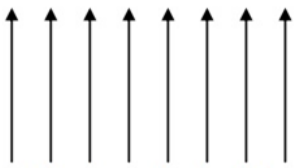
Examples: Manganese sulphate, ferric oxide, ferrous sulphate, nickel sulphate, etc.

FERROMAGNETIC MATERIALS

Certain materials like iron, cobalt, nickel and certain alloys exhibit high degree of magnetization. These materials show spontaneous magnetization. (i.e) they have small amount of magnetization even in the absence of external magnetic field. This indicates that there is strong internal field within the material which makes atomic magnetic moments with each other. This phenomenon is known as ferromagnetism.

Properties of ferromagnetic materials:

- All the dipoles are aligned parallel to each other due to the magnetic interaction between the two dipoles.
- They have permanent dipole moment. They are strongly attracted by the magnetic field.
- They exhibit magnetization even in the absence of magnetic field. This property of ferromagnetic material is called as spontaneous magnetization.
- They exhibit hysteresis curve.
- On heating, they lose their magnetization slowly. The dipole alignment is shown in fig.



- The susceptibility is very high and depends on the temperature. It is given by

$$\chi = \frac{C}{T-\theta} \text{ (for } T>\theta\text{) paramagnetic behaviour}$$

(for $T<\theta$) ferromagnetic behavior
where C is the Curie constant and θ is the paramagnetic Curie temperature.

ANTIFERROMAGNETIC MATERIALS

Antiferromagnetic materials are magnetic materials which exhibit a small positive susceptibility of the order of 10^{-3} to 10^{-5}. In antiferromagnetic materials, the susceptibility increases with increasing temperature and it reaches maximum at a certain temperature called Neel Temperature, T_N. With further increase in temperature, the material reaches the paramagnetic state. The material is antiferromagnetic below T_N.

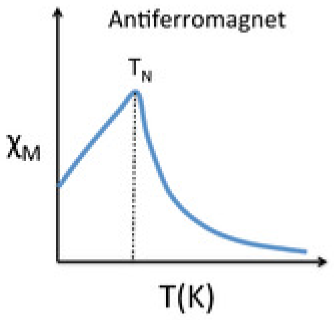


Fig 2.

Properties of antiferromagnetic materials

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- The electron spin of neighboring atoms are aligned antiparallel. (i.e) the spin alignment is antiparallel.

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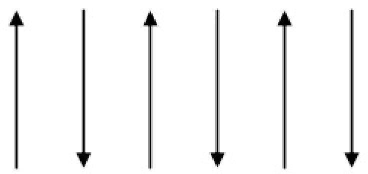


- Antiferromagnetic susceptibility is mainly depends on temperature.
- The susceptibility of the antiferromagneitc material is small and positive. It is given by

$$\chi = \frac{C}{T+\theta} \text{ when } T>T_N$$

$$\chi \propto T \quad \text{when } T<T_N$$

- The susceptibility initially increases slightly with the temperature and beyond Neel temperature, the susceptibility decreases with temperature.

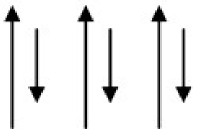
FERRIMAGNETIC MATERIALS

Properties of ferrites

- Ferrites have net magnetic moment.
- Above Curie temperature, it becomes paramagnetic, while it behaves ferromagnetic material blow Curie temperature.
- The susceptibility of ferrite is very large and positive. It is temperature dependent and is given by

$$\chi = \frac{C}{T\pm\theta} \text{for } T>T_N$$

- Spin alignment is antiparallel of different magnitudes as shown fig.



- Mechanically it has pure iron character.
- They have high permeability and resistivity.
- They have low eddy current losses and low hysteresis losses.

Molecular field theory on ferromagnetism

The transition metals Fe, Co and Ni exhibit magnetisation even when the magnetising field is removed. This phenomenon is called ferromagnetism. Weiss postulated the existence of an internal molecular field ‘Hi’ which favours spontaneous magnetisation of a ferromagnetic material and is proportional to the magnetisation I of the material. This magnetisation ‘I’ is called spontaneous magnetisation (magnetisation arising in the absence of any applied field).

∴ $Hi = \lambda I$ where λ is molecular field coefficient or Weiss constant. The net effective magnetic field is equal to H+Hi.

$$\therefore He = H + Hi = H + \lambda I$$

$$\therefore I = \frac{\mu_{0N\mu^2}}{3kT}(H + \lambda I)$$

$$\chi = \frac{I}{H} = \frac{\mu_{0N\mu^2}}{3kT} + \lambda \frac{I}{H} \frac{\mu_{0N\mu^2}}{3kT}$$

$$\chi\left(1 - \frac{C\lambda}{T}\right) = \frac{C}{T} \text{ where } C = \frac{\mu_{0N\mu^2}}{3k}$$

$$\therefore \chi = \frac{C}{T - \lambda C} = \frac{C}{T - \theta} \text{ where } \lambda C = \theta$$

This is called Curie-Weiss Law and θ is called Curie temperature. When T = θ, χ→ ∞ and for temperatures less than θ, the relation does not signify any meaning. This means, there exists spontaneous magnetisation even in the absence of the external magnetic field. The material is ferromagnetic below Curie temperature, 'θ' and becomes paramagnetic above Curie temperature.

Thermal agitation opposes the tendency of Weiss molecular field to align the molecular magnets. But below Curie temperature Weiss field energy overcomes the thermal energy. Consequently alignments of most molecular magnets results giving rise to magnetisation of the material even in the absence of applied field called phenomenon of spontaneous magnetisation.

Temperature dependence of spontaneousmagentisation

From the experimental curve, when T= 0, $\frac{I}{I_S}$=1.

That is the spontaneous magnetisation is maximum.

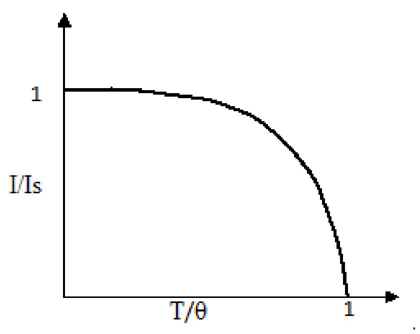


Fig.3 Temperature dependence of spontaneous magnetisation

Where I = spontaneous magnetisation and Is= saturation magnetisation; When T= θ, I/Is= 0 and the spontaneous magnetisation vanishes.

When temperature is low, Weiss field overcomes the thermal energy and T= 0 give rise te maximum magnetisation. But as the temperature increases thermal energy increases which randomises more and more of the parallel spins and at Curie temperature, all the parallel alignment of spins vanishes resulting in the zero value of spontaneous magnetisation. The substance then becomes paramagnetic.

DOMAIN THEORY OF FERROMAGNETISM

We can observe that ferromagnetic materials such as iron does not have magnetization unless they have been previously placed in an external magnetic field. But according to Weiss theory, the molecular magnets in the ferromagnetic material are said to be aligned in such way that, they exhibit magnetization even in the absence of external magnetic field. This is called spontaneous magnetization. (i.e) it should have some internal magnetization due to quantum exchange energy.

According to Weiss hypothesis, a single crystal of ferromagnetic material is divided into large number of small regions called domains. These domains have spontaneous magnetization due to the parallel alignment of spin magnetic moments in each atom. But the direction of spontaneous magnetization varies from domain to domain and is oriented in such way that the net magnetization of the specimen is zero

The boundaries separating the domains are called domain walls. These domain walls are analogous to the grain boundaries in a polycrystalline material.

DOMAIN GROWTH

Now when the magnetic field is applied, then the magnetization occurs in the specimen by two ways

1.By motion of domain walls
2. By rotation of domain walls

By motion of domain walls

The motion of domain walls takes place in weak magnetic fields. Due to this weak field applied to the specimen the magnetic moment increases and hence the boundary of domains displaced, so that the volume of the domains changes as shown in fig (4).

By rotation of domain walls

The rotation of domain walls takes place in strong magnetic fields. When the external field is high then the magnetization changes by means of rotation of the direction of magnetization towards the direction of the applied field as shown fig(4).

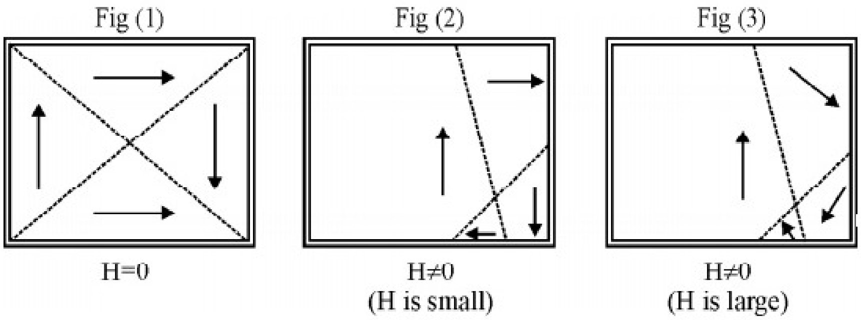


Fig 4. Domain theory of ferromagnetism

Energies involved in the domain growth (or) Origin of Domain theory of Ferromagnetism

We can understand the origin of domains from the thermodynamic principle i.e., in equilibrium, the total energy of the system is minimum. To study the domain structure clearly, we must know four types of energy involved in the process of domain growth.

Exchange energy (or) Magnetic field energy.

Crystalline energy (or) Anisotropy energy.

Domain wall energy (or) Bloch wall energy.

Magnetostriction energy

1. Exchange energy (or) Magnetic Field energy

The interaction energy which makes the adjacent dipoles align themselves is the called **exchange energy (or) magnetic field energy.**

The interaction energy makes the adjacent dipoles align themselves. It arises from interaction of electron spins. It depends upon the interatomic distance. This exchange energy also called magnetic field energy is the energy required in assembling the atomic magnets into a single domain and this work done is stored as potential energy. The size of the domains for a particular domain structure may be obtained from the principle of minimum energy. The volume of the domain may vary between say, say, 10^{-2} to 10^{-6} cm^3.

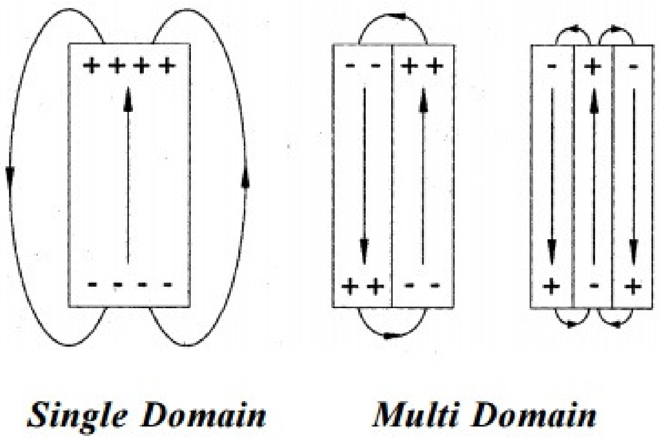


Fig.5 Exchange energy in ferromagnetism

2.Anisotropy energy

The excess energy required to magnetize a specimen in particular direction over that required to magnetize it along the easy direction is called the **crystalline anisotropy energy.** In ferromagnetic materials there are two types of directions of magnetization namely,

Easy direction and

Hard directions.

In easy direction of magnetization, weak field can be applied and in hard direction of magnetization, strong field should be applied. For producing the same saturation magnetization along both hard and easy direction, strong fields are required in the hard direction than the easy direction. For example in iron easy direction is [100], medium direction is [110] and the hard direction is [111] and it is shown in fig. From the fig we can see that very strong field is required to produce magnetic saturation in hard direction [111] compared to the easy direction [100].

Therefore the excess of energy required to magnetize the specimen along hard direction over that required to magnetize the specimen along easy direction is called crystalline anisotropy energy.

In easy direction of magnetization, weak field can be applied and in hard direction of magnetization, strong field should be applied. As shown in figure magnetization curves for iron with the applied field along different crystallographic direction crystallographic directions have been drawn. For example, in BCC iron the easy direction is [100], the medium direction is [110], and the hard direction [111]. The energy difference between hard and easy direction to magnetize the material is about. This energy is very important in determining the characteristic domain boundaries.

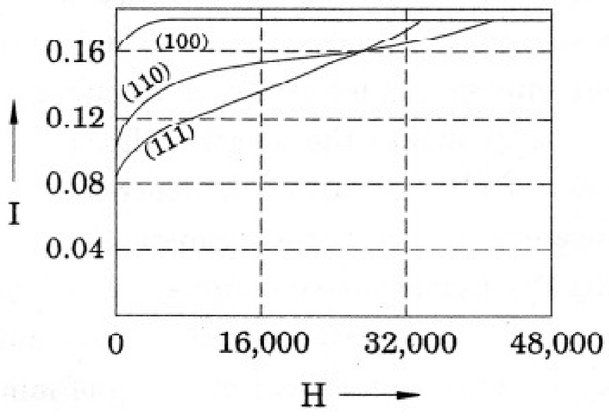


Fig 6 Anisotropy energy in ferromagnetism

3. Domain wall energy or Bloch wall energy

Domain wall is a transition layer which separates the adjacent domains, magnetized in different directions. A thin boundary or region that separates adjacent domains magnetized in different directions is called domain wall or Bloch wall.

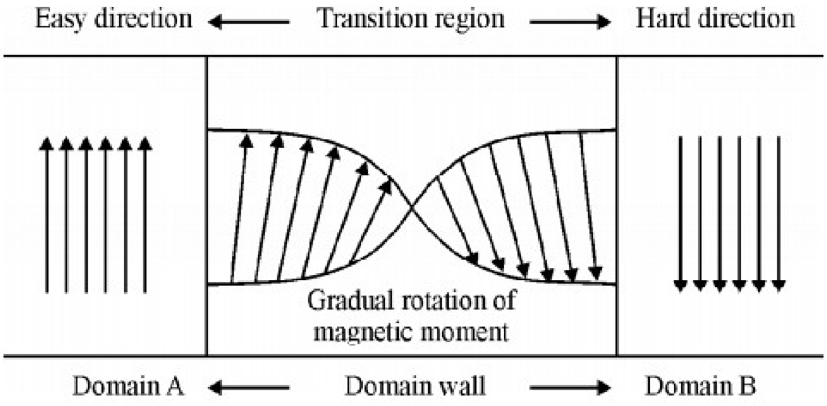


Fig.7 The change of electron spin in the transition region of Bloch wall

The size of the Bloch walls is about 200 to 300 lattice constant thickness. In going from one domain to another domain, the electron spin changes gradually as shown in figure. The energy of domain wall is due to both exchange energy and anisotropic energy.

Based on the spin alignments, two types of Bloch walls may arise, namely

Thick wall: When the spins at the boundary are misaligned and if the direction of the **spin changes gradually** as shown figure, it leads to a thick Bloch wall. Here the misalignments of spins are associated with exchange energy.

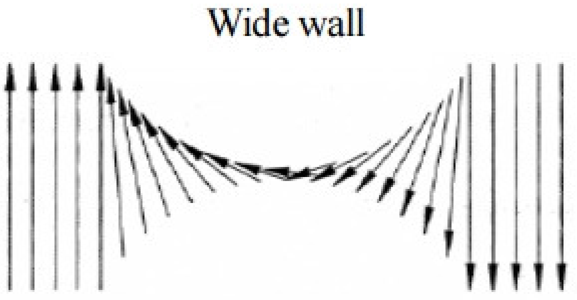


Fig.8 The change of electron spin in the transition region of thick wall

Thin wall: When the spins at the boundaries **changes abruptly,** then the anisotropic energy becomes very less. Since the anisotropic energy is directly proportional to the thickness of the wall, this leads to a thin Bloch wall.

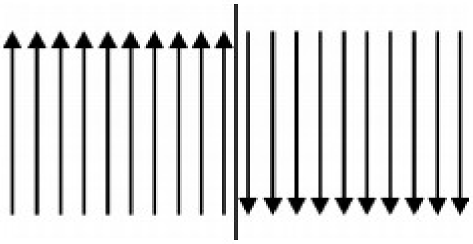


Fig.9 The change of electron spin in the transition region of thin wall

4. Magnetostriction energy

When a material is magnetized, it is found that it suffers a change in dimensions. This phenomenon is known as **Magnetostriction.** This deformation is different along different crystal directions. So if the domains are magnetized in different directions, they will either expand or shrink. This means that work must be done against the elastic restoring forces. The work done by the magnetic field against these elastic restoring forces is called magneto-elastic energy or Magnetostrictive energy.

EXPLANATION OF HYSTERESIS

When a ferromagnetic material is made to undergo through a cycle of magnetization, the variation of magnetic induction (B) with respect to applied field (H) can be represented by a closed hysteresis loop (or) curve. (i.e) it refers to the lagging of magnetization behind the magnetizing field.If magnetizing field (H) is applied to a ferromagnetic material and if H is increased to Hmax the material acquires magnetism. So the magnetic induction also increases, represented by oa in the fig.10.

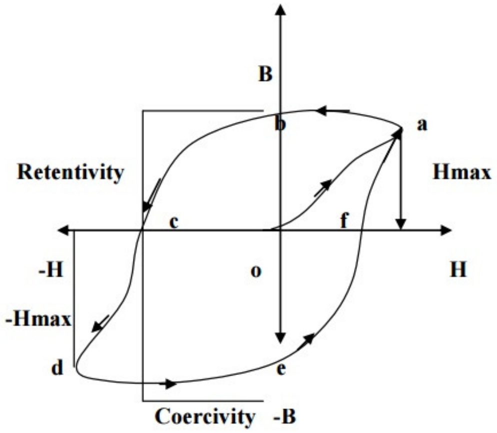


Fig. 10 Hysteresis curve

Now if the magnetic field is decreased from Hmax to zero, the magnetic induction will not fall rabidly to zero, but falls to ‘b’ rather than removed, the material still acquire some magnetic induction (ob) which is so called residual magnetism or retentivity.

Now, to remove the residual magnetism, the magnetic field strength is reversed and increased to –Hmax represented as ‘oc’ so called coercivity(-H) is reduced to zero and the corresponding curve ‘defa’ is obtained.

Retentivity (or) Remanence

When the external magnetic field is applied to a magnetic material is removed, the magnetic material will not loss its magnetic property immediately. There exits some residual intensity of magnetization in the specimen even when the magnetic field is cut off. This is called residual magnetism (or) retentivity.

Coercivity

The residual magnetism can be completely removed from the material by applying a reverse magnetic field. Hence coercivity of the magnetic material is the strength of reverse magnetic field (-Hc) which is used to completely demagnetize the material.

HYSTERESIS ON THE BASIS OF DOMAIN THEORY

We know when the ferromagnetic material is subjected to external field, there is an increase in the value of the magnetic moment due to two process.

1 .The movement of domain walls

2. Rotation of domain walls

When small external field is applied, the domains walls displaced slightly in the easy direction of magnetization. This gives rise to small magnetization corresponding to the initial portion of the hysteresis curve (OA) as shown in figure. Now if the field is removed, then the domains returns to the original state, and is known as reversible domains.

When the field is increased, large numbers of domains contribute to the magnetization and thus the magnetization increases rabidly with H. Now, even when the field is removed, because of the displacement of domain wall to a very large distance, the domain boundaries do not come back to their original position. This process is indicating as AB in figure and these domains are called irreversible domains.

Now, when the field is further increased, the domains starts rotating along the field direction and the anisotropic energy stored in the hard direction, represented as BC in the figure. Thus the specimen is said to attain the maximum magnetization. At this position, even when the field is removed the material possess maximum magnetization, called residual magnetism or retentivity, represented by OD in fig.11

Actually after the removal of external field, the specimen will try to attain the original configuration by the moment of domain wall. But this moment is stopped due to presence of impurities, lattice imperfections etc. therefore to overcome this; a large amount of reverse magnetic field is applied to the specimen. The amount of energy spend to reduce the magnetization to zero is called coercivity represented by OE in the fig.11

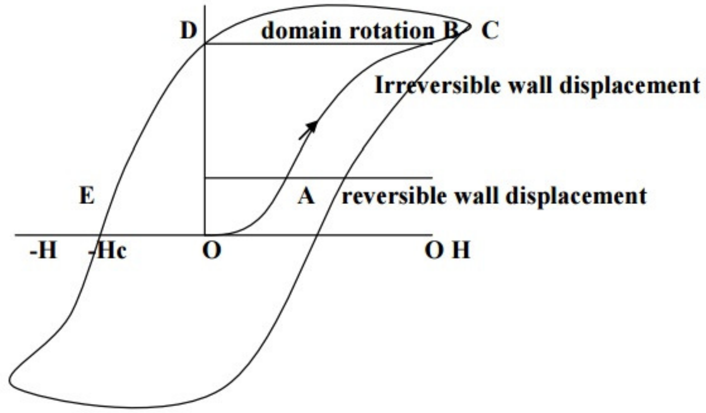


Fig. 11

It is the loss of the energy in taking a ferromagnetic specimen through a complete cycle of magnetization and the area enclosed is called hysteresis loop.

SOFT AND HARD MAGNETIC MATERIALS

Depending upon the direction of magnetization by external field, and the area of hysteresis, magnetic can be classified into two types as,

TYPES OF MAGNETIC MATERIALS

Magnetic materials are classified onto two types.

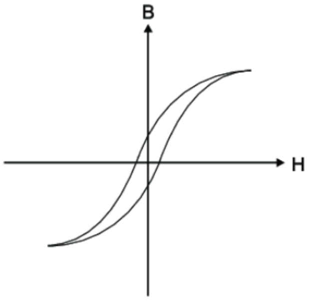
1. Soft magnetic materials
2. Hard magnetic materials

Soft magnetic materials:

Materials which are easy to magnetize and demagnetize are called soft magnetic materials.

Example –pure iron, cast iron, carbon steel, silicon steel, mumetal.

1. The magnetic materials can be easily magnetize and demagnetize.
2. The have high permeability.
3. Magnetic energy stored is not high.
4. Low hysteresis loss due to small hysteresis loop area.
5. Coercivity and retentivity are small.
6. The eddy current loss is small due to its high resistivity.
7. The domain walls are easy to move.
8. They are used in electric motor, generators, transformers, relays, telephone receivers, radar.

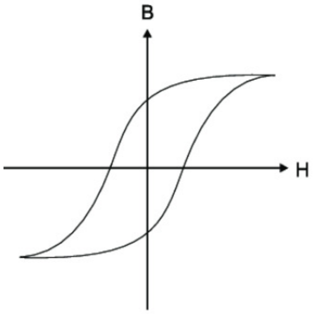


Soft Magnetic Materials

Hard magnetic materials:

Materials which retain their magnetism and are difficult to demagnetize are called hard magnetic materials.Example –tungsten steel, cobalt steel, alini, alnico, hypernic

1. The magnetic materials can not be easily magnetize and demagnetize.
2. The have low permeability.
3. Magnetic energy stored is high.
4. Large hysteresis loss due to large hysteresis loop area.
5. Coercivity and retentivity are large.
6. The eddy current loss is high due to its low resistivity.
7. The movement of domain wall must be prevented.
8. They are used in loud speakers and electrical measuring instruments.



Hard Magnetic Materials

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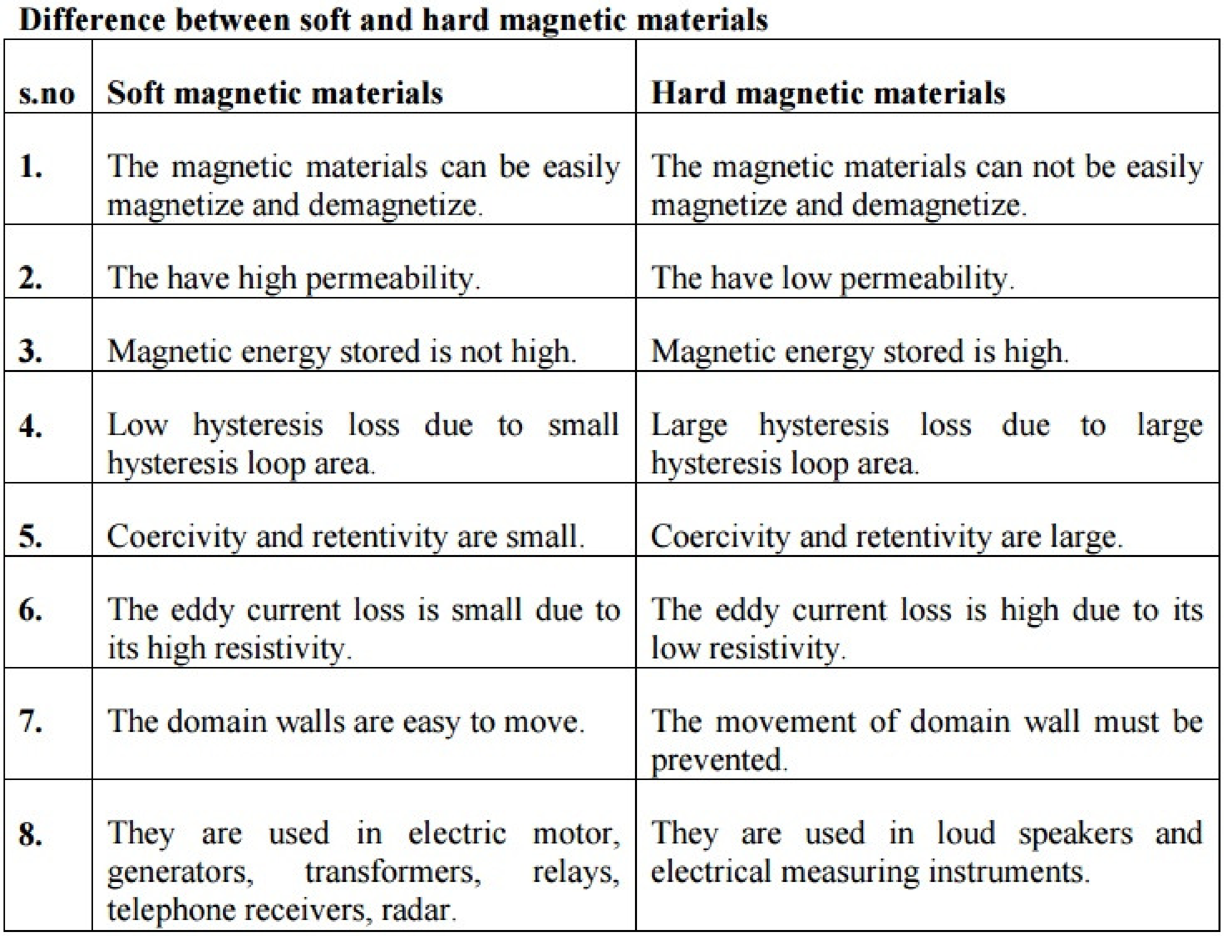

Difference between soft and hard magnetic materials

s.no	Soft magnetic materials	Hard magnetic materials
1.	The magnetic materials can be easily magnetize and demagnetize.	The magnetic materials can not be easily magnetize and demagnetize.
2.	The have high permeability.	The have low permeability.
3.	Magnetic energy stored is not high.	Magnetic energy stored is high.
4.	Low hysteresis loss due to small hysteresis loop area.	Large hysteresis loss due to large hysteresis loop area.
5.	Coercivity and retentivity are small.	Coercivity and retentivity are large.
6.	The eddy current loss is small due to its high resistivity.	The eddy current loss is high due to its low resistivity.
7.	The domain walls are easy to move.	The movement of domain wall must be prevented.
8.	They are used in electric motor, generators, transformers, relays, telephone receivers, radar.	They are used in loud speakers and electrical measuring instruments.

Energy product

It is the product of residual magnetism B_r and coercivity which H_C gives the maximum amount of energy stored in the specimen.

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Energy product = $B_r \times H_C$

Hysteresis loss

When the specimen is taken through a cycle of magnetization. There is a loss of energy in the form of heat. This loss of energy is known as Hysteresis loss.

DIELECTRIC MATERIALS

INTRODUCTION

Solids which have an energy gap of three eV or more are termed as insulators. In these materials, it is almost not possible to excite the electrons from the valence band to conduction band by an applied field. Generally dielectrics are also called as insulators, thereby poor conductors of electricity. However they allow movement of some electrons at abnormally high temperatures, causing a small flow of current.

Dielectrics are non-metallic materials of high specific resistance ρ, negative temperature coefficient of resistance (-α), large and insulation resistance. Insulation resistance will be affected by moisture, temperature, applied electric field and age of dielectrics.

PROPERTIES

- Generally, the dielectrics are non-metallic materials of high resistivity.
- They have a very large energy gap (more than 3eV).
- All the electrons in the dielectrics are tightly bound to their parent nucleus.
- As there are no free electrons to carry the current, the electrical conductivity of dielectrics is very low.
- They have negative temperature coefficient of resistance and high insulation resistance.

Dielectric materials are electrically non-conducting materials such as glass, ebonite, mica, rubber, wood and paper. All dielectric materials are insulating materials. The difference between a dielectric and an insulator lies in their applications.

If the main function of non-conducting material is to provide electrical insulation, then they are called as insulator. On the other hand, if the main function of non-conducting material is to store electrical charges then they are called as dielectrics.

Electrostatics

Electric charge is a fundamental property of matter and charge exist in integral multiple of electronic charge. Electrostatics can be defined as the study of electric charges at rest. Electric fields have their sources in electric charges. The two fundamental laws governing the electrostatic fields, viz, (1) Coulomb's Law and (2) Gauss's Law. Both these law have experimental basis. Coulomb's law is applicable in finding electric field due to any charge distribution, Gauss's law is easier to use when the distribution is symmetrical.

Coulomb's Law

Coulomb's Law states that the force between two point charges Q_1 and Q_2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

Point charge is a hypothetical charge located at a single point in space. It is an idealised model of a particle having an electric charge.

Mathematically, $F = \frac{kQ_1Q_2}{R^2}$, where k is the proportionality constant.

In SI units, Q_1 and Q_2 are expressed in Coulombs(C) and R is in meters.

Force F is in Newtons (N) and $k = \frac{1}{4\pi\varepsilon_0}$, ε_0 is called permittivity of free space.

The charges are in free space. If the charges are any other dielectric medium, we will use $\varepsilon = \varepsilon_0\varepsilon_r$ instead where ε_r is called the relative permittivity or the dielectric constant of the medium.

Electromagnetic Waves

Electromagnetic theory is a discipline concerned with the study of charges at rest and in motion. Electromagnetic principles are fundamental to the study of electrical engineering and physics. Electromagnetic theory is also indispensable to the understanding, analysis and design of various electrical, electromechanical and electronic systems.

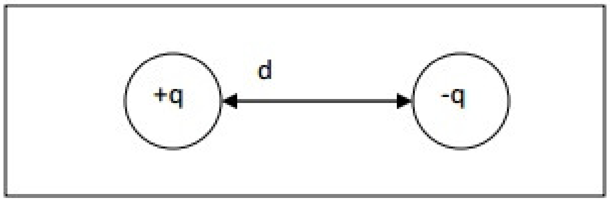
Electromagnetic waves or EM waves are waves that are created as a result of vibrations between an electric field and a magnetic field. In other words, EM waves are composed of oscillating magnetic and electric fields. Electromagnetic waves are formed when an electric field comes in contact with a magnetic field. They are hence known as ‘electromagnetic’ waves. The electric field and magnetic field of an electromagnetic wave are perpendicular (at right angles) to each other. They are also perpendicular to the direction of the EM wave.

EM waves travel with a constant velocity of 3.00 x 108 ms-1 in vacuum. They are deflected neither by the electric field, nor by the magnetic field. However, they are capable of showing interference or diffraction. An electromagnetic wave can travel through anything - be it air, a solid material or vacuum. It does not need a medium to propagate or travel from one place to another. Mechanical waves (like sound waves or water waves), on the other hand, need a medium to travel. EM waves are 'transverse' waves. This means that they are measured by their amplitude (height) and wavelength (distance between the highest/lowest points of two consecutive waves).

FUNDAMENTAL DEFINITIONS AND PROPERTIES

ELECTRIC DIPOLE

A system consisting of two equal and opposite charges n(+q, -q) separated by a distance (d) is called an electric dipole.



DIPOLE MOMENT (μ)

The product of the magnitude of the charge (q) and distance between two charges (d) is called as dipole moment.

Dipole moment = qd (coulomb-metre)

PERMITTIVITY (ε)

The permittivity represents the dielectric property of a medium. It indicates easily polarisable nature of material. Its unit is farad/metre

DIELECTRIC CONSTANT (ε_r)

A dielectric characteristic of a material is determined by its dielectric constant. It is a measure of polarisation of the dielectrics.

Definition

It is the ratio between absolute permittivity of the medium (ε) and permittivity of free space (εo).

Dielectric constant = Absolute permittivity (ε) / Permittivity of free space (ε_o)

$$\varepsilon_r = \varepsilon_o / \varepsilon$$

POLARIZATION

Definition

The process of producing electric dipoles inside the dielectric by the application of an external electrical field is called polarization in dielectrics.

POLARISABILITY (α)

It is found that the average dipole moment field (E).

$$\mu \quad \alpha \; E$$

or $\mu = \alpha E$

Where (α) is the polarisability.

$$\alpha = \mu / E$$

Polarisability is defined as the ratio of average dipole moment to the electrical field applied. Its unit is farad m^2.

Polarisation vector [P]

It is defined as the average dipole moment per unit volume of a dielectric. If N is the number atoms per unit volume of a dielectric and (μ) is average dipole moment per atom, then

$$\vec{P} = N\mu \quad (\text{coulomb}/m^2)$$

An understanding of the interaction of electromagnetic radiation with tissue is important for many reasons, apart from its intrinsic interest. It underpins many imaging techniques, and it is essential to an understanding of the detection of electrical events within the body, and the effect of externally applied electric currents.

TISSUE AS A LEAKY DIELECTRIC

If two electrodes are placed over the abdomen and the electrical impedance is measured between them over a wide range of frequencies the he results obtained might be as shown in fig.12. The results will depend upon the type and size of electrodes, particularly at the lowest frequencies, exactly where the electrode placed. However the result is mainly a function of the tissue properties. The impedance always drops with increasing frequency.

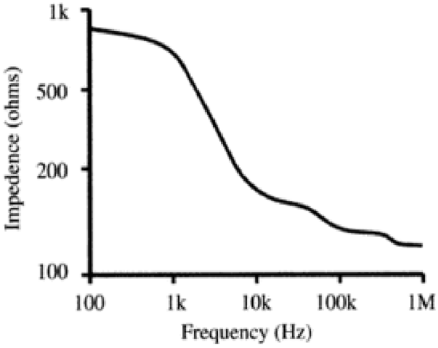


Fig.12. Electrical impedance as a function of frequency for two electrodes placed on the abdomen

We are familiar with the concept of conductors, which have free charge carriers, and of insulators, which have dielectric properties as a result of the movement of bound charges under the influence of an applied electric field. Tissues though contain both free and bound charges, and thus exhibit simultaneously the properties of a conductor and a dielectric. For tissue the conductivity has to be included to account for the redistribution of bound charges in the dielectric. If tissue is considered as a dielectric, the term permittivity has to be included to account for the movement of free charges.

The interaction between electromagnetic waves and tissue can be explored from the properties of dielectrics. The dielectrics are used in insulators in cables and electronic components. A primary requirement of these dielectrics is that their conductivity is very low ($< 10^{-10}$ Sm^{-1}). Metals and alloys in which the conduction is by free electrons, have high conductivities ($>10^{4}$ Sm^{-1}). Intermediate between metals and insulators are semiconductors (conduction by excitation of holes and electrons) with conductivities in the range 10^{0}-10^{-4} Sm^{-1}, and electrolytes (conduction by ions in solution) with conductivities of the order of 10^{0}-10^{2} Sm^{-1}. Tissue can be considered as a collection of electrolytes contained within membranes of assorted dimensions. None of the constituents of tissue can be considered to have 'pure'resistance or capacitance-the two properties are inseperable.

Consider slabs of an ideal conductor and an ideal insulator, each with surface area A and thickness x (fig). If the dielectric has relative permittivity ε_r then the slab has the capacitance $C = \varepsilon_0\varepsilon_r A/x$. The conductance of the slab is $G=\sigma A/x$, where the conductivity is σ. Also conductivity σ is the current density due to unit applied electric field (from $J=\sigma E$), and the permittivity of free space ε_0 is the charge density due to unit electric field, from Guass' law. The relative permittivity $\varepsilon_r=C_m/C_0$, where C_0 is the capacitance of a capacitor in vacuo, and C_m is the capacitance with a dielectric completely occupying the region containing the electric field. In tissue, both of these properties are present, so a model capacitor with a parallel conductance is as shown in the fig.

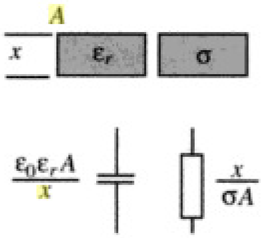


Fig.13 Slabs of an insulator,on the left, and a conductor on the right. The capacitance of the insulator and the resistance of the conductor are given.

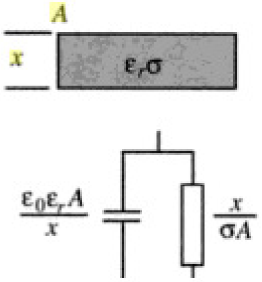


Fig.14 Tissue with both capacitive and resistive properties in parallel. The capacitance and resisitance of the two arms are marked.

The equations $C = \varepsilon_0\varepsilon_r A/x$ and $G = \sigma A/x$ define the static capacitance and conductance of the dielectric i.e., the capacitance and conductance at zero frequency. If we apply an alternating voltage to the real dielectric, the current will lead the voltage.

Clearly if $G = 0$, the phase angle $\theta = \pi/2$, i.e. the current leads the voltage by $\pi/2$, for pure capacitance. If $C= 0$, current and voltage are in phase, as expected for pure resistance. For real dielectric theadmittance is given by $Y^* = G+j\omega C$, where the * convention has been used to denote a complex variable.

For convenience, consider generalized permittivity $\varepsilon^* = \varepsilon' - j\varepsilon''$ which includes the effect of both the resistive and capacitive elements in real dielectric.

ε' is the real part and ε'' is the imaginary part.

$\therefore$ By relating the generalized permittivit to the model of the real dielectric, the admittance is given by

$$Y^* = G+j\omega C = \frac{A}{x}(\sigma+j\omega\varepsilon_0\varepsilon_r)$$

By analogy with an ideal capacitance C which has admittance $j\omega C$, we can define the complex capacitance C^* of the real dielectric,

$$C^* = \frac{Y^*}{j\omega} = \frac{A}{x}\left(\frac{-j\sigma}{\omega} + \varepsilon_0\varepsilon_r\right) = \frac{A}{x}\ \varepsilon_0\varepsilon^* = \varepsilon^* C$$

i.e. $\varepsilon^* = \varepsilon_r - \frac{-j\sigma}{\omega\varepsilon_0}$

thus $\varepsilon' = \varepsilon_r$ and $\varepsilon'' = \dfrac{\sigma}{\omega\varepsilon_0}$

From this it can be seen that we can consider the properties of our non-ideal capacitor as being the result of inserting a dielectric with a relative permittivity ε* in an ideal capacitor C. The real part ε' is the relative permittivity ε_r of the ideal capacitor, and the imaginary part jε" is associated with the resistive properties.

We can also consider the admittance in terms of complex conductivity,

$$Y^* = G + j\omega C = \frac{A}{x}(\sigma + j\omega\varepsilon_0\varepsilon_r) = \frac{A}{x}\sigma^*$$

i.e. $\sigma^* = \sigma + j\omega\varepsilon_0\varepsilon_r$

The complex permittivity and complex conductivity are related by

$$\sigma^* = j\omega\varepsilon^*\varepsilon_0$$

Thus the behavior of the conductivity and permittivity can be related. Also as the frequency tends to zero, the complex conductivity becomes purely real, and in the high frequency limit, the complex permittivity becomes purely real. Thus the conductivity is dominant at low frequencies and the permittivity is dominant at high frequencies.

RELAXATION PROCESSES

If there is a redistribution of charges within the dielectric as a result of applying the electric field, the response to a step funtion of the electric field will be a funtion of time, and the equation for the capacitance C and conductance G will only be true in the limit as time goes to infinity. In other words ε and σ are a function of frequency, and have a characteristic time constant which is called a relaxation time. There are a number of models of relaxation processes, all of which give qualittevely similar results.

DEBYE MODEL

When a capacitor is charged, the applied electric field E either creates or orienttes electricla dipoles within the material. This process is called polarization. Assuming that the process of polarization has an exponnential appraoch to its final value it can be shown that the real and imaginary partsof the complex permittivity will be given by

$$\varepsilon^* = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{1 + j\omega\tau}$$

Now $\varepsilon^* = \varepsilon' - j\varepsilon''$, so by equating real and imaginary parts we find

$$\varepsilon^* = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{1+\omega^2\tau^2}$$

$$\varepsilon'' = \frac{(\varepsilon_s - \varepsilon_\infty)\omega\tau}{1+\omega^2\tau^2}$$

where ε_∞ and ε_s are the permittivities at infinite and zero (static) frequencies, respectively. These two equations are the Debye equations. The real and imaginary parts of the complex permittivity are plotted in fig.16

The same results would be obtained for an equaivalent circuit as shown in fig.15

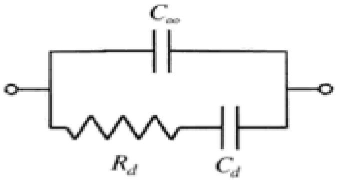
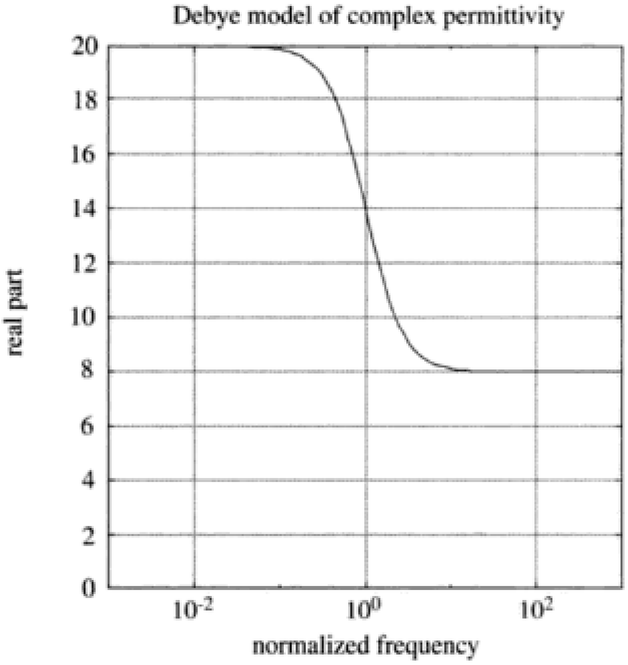


Fig.15 Electrical circuit equivalent of the Debye model



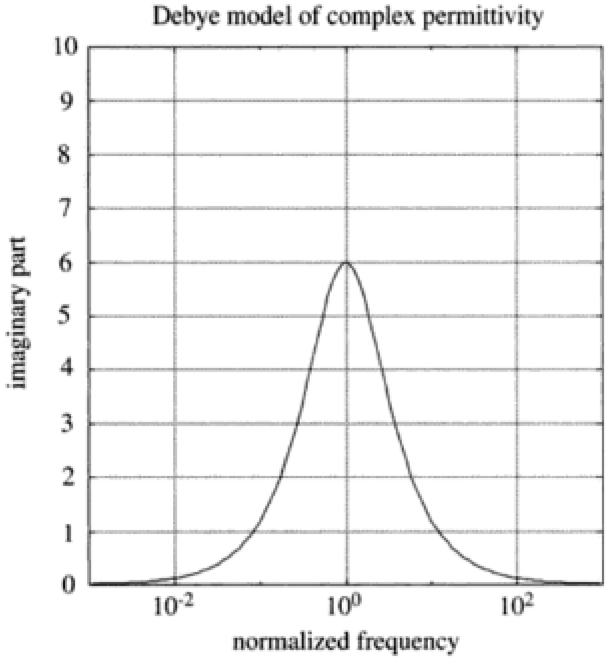


Fig.16 The real(top) and imaginary(bottom) parts of the complex permittivity given by the Debye equations.

$$C_{\infty} = \frac{\varepsilon_0 \varepsilon_{\infty}}{k}$$

$$R_d = \frac{k\tau}{\varepsilon_0 (\varepsilon_s - \varepsilon_{\infty})}$$

$$C_d = C_s - C_{\infty} = \varepsilon_0 \frac{\varepsilon_s - \varepsilon_{\infty}}{k} \text{ (k is a geometrical factor)}$$

However when measurements of the relative permittivity aremade over wide range of frequencies, it is found that the results do not agree with the predictions from the Debye model. In particular, the measured values spread over a wider range of frequencies, the value of ε” is too low. Cole and Cole introduced an empirical modification to match the models to the data.

Cole-Cole Model

The two figures given in fig.17 show the real and imaginary partsa of a relaxation process. The full curves with the steeper gradients are the prediction from the Debye model. The less steep curves show a typical measurement from actual tissue. The agreement between theoretical and measured values is not very good. Cole and Cole proposed an alternative formula which was similar to the Debye equations but had an additional term called alpha.

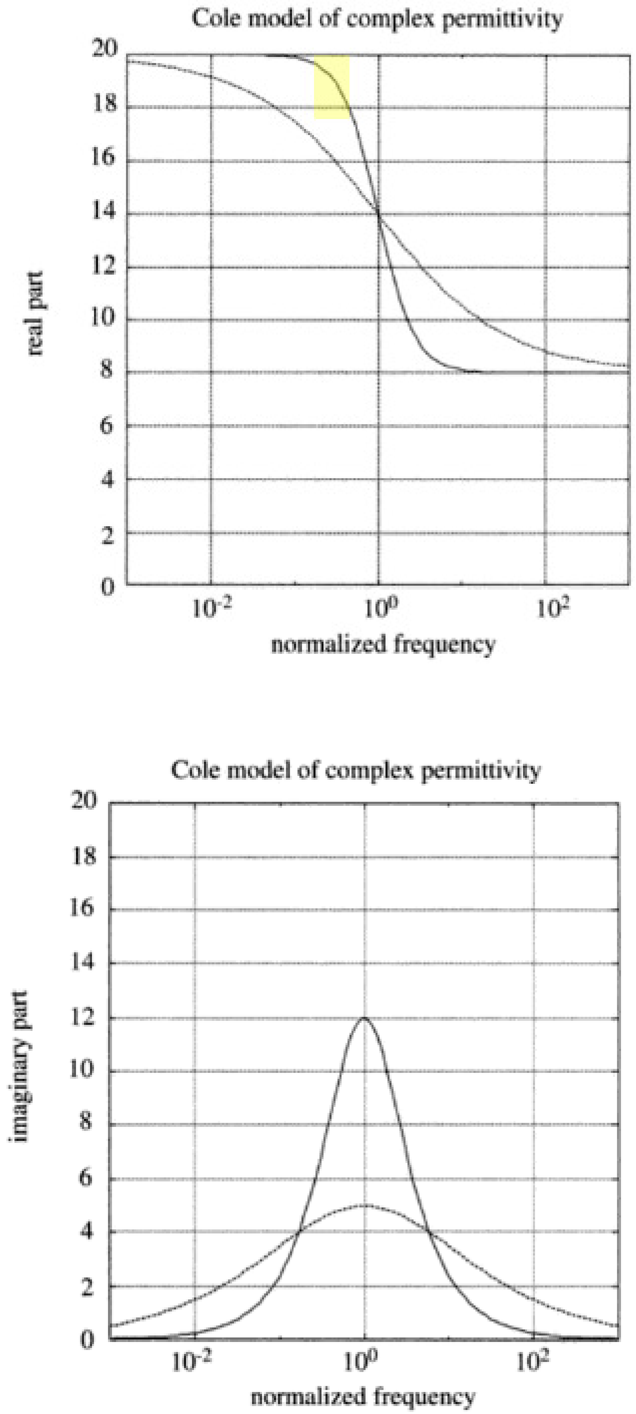


Fig.17 The real(top) and imaginary(bottom) parts of the Cole and Debye models for a relaxation process. The less steep curves are for the Cole model and agree well with many measurements made on biological tissues.

Cole proposed that the realdistribution could be modelled by

$$\varepsilon^{*} - \varepsilon_{\infty} = \frac{(\varepsilon_{s} - \varepsilon_{\infty})}{1 + j\omega\tau^{1-\alpha}}$$

Alpha can be chosen to give a good fit to measured data. It can be seen that, if α = 0, the equation reduces to the Debye model.

The real and imaginary components of the complex permittivity are given by the equations, which reduce to the values derived for the Debye model when α = 0:

$$\varepsilon' = \varepsilon_{\infty} + \frac{(\varepsilon_{s} - \varepsilon_{\infty})[1 + (\omega\tau)^{1-\alpha}\sin(\alpha\pi/2)}{1 + 2(\omega\tau)^{1-\alpha}\sin(\alpha\pi/2) + (\omega\tau)^{2(1-\alpha)}}$$

$$\varepsilon' = \varepsilon_{\infty} + \frac{(\varepsilon_{s} - \varepsilon_{\infty})(\omega\tau)^{1-\alpha}\cos(\alpha\pi/2)}{1 + 2(\omega\tau)^{1-\alpha}\sin(\alpha\pi/2) + (\omega\tau)^{2(1-\alpha)}}$$

Hence Cole –Cole model is a model of the measured data. It has been applied very successfully to a wide variety of materials.

Cole equation can be written in terms of impedance instead of a complex permittivity. The impedance Z is given by

$$Z = R_{\infty} + \frac{R_{0} - R_{\infty}}{1 + (jf/f_{c})^{`1-\alpha}}$$

Where R_0 and R_∞ are the resistances at zero frequency (i.e. DC) and infinity, respectively. f_c is the characteristic frequency. It should be emphasized that the characteristic frequency is not the same when the analysis is carried out in terms of the complex permittivity.

PROBLEMS

1. A paramagnetic material has a magnetic field intensity of 10^4 $\mathrm{Am^{-1}}$. If the susceptibility of the material at room temperature is 3.7×10^{-5}. Calculate the magnetization and flux density in the material.

Solution:

Susceptibility (i) $\chi = \dfrac{I}{H}$

$\therefore$ Intensity of Magnetization $I = \chi H$

$= 3.7 \times 10^{-3} \times 10^4$

$I = 37\ \mathrm{Am^{-1}}$

(ii) Flux density

$B = \mu_0 (H+I)$

$B = 4\pi \times (10^4+37)$

$B = 0.012612\ \mathrm{Wb/m^2}$

2. A magnetic field of 2000 A/m is applied to a material which has a susceptibility of 1000. Calculate (i) Intensity of magnetization and (ii) Flux density.

Solution: Given data: H= 2000 A/m

$\chi = 1000$

$I = ?$

$B = ?$

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(i) Intensity of magnetization $I = \chi H$

$I = 1000 \times 2000$

$\therefore$ Intensity of magentization $I = 2 \times 10^6$ A/m

(ii) Flux density $B = \mu H$

Here $\mu = 1 + \chi$

$\therefore \mu = 1+1000$

$\therefore B = 1001 \times 2000 \times 4\pi \times 10^{-7}$

$\therefore$ Flux density $B = 2.5145$ Wb/m^2

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Part A Questions

1. Write some properties of dia, para and ferro magnetic materials.
2. What is meant by domain and domain wall?
3. What is Bohr magneton?
4. What is Curie constant? What is Curie Law?
5. Mention the properties of hard and soft magnetic materials.
6. Classify the magnetic materials based on their magnetic moments.
7. What is meant by hysteresis loop and what do you infer from it?
8. Difference between soft and hard magnetic materials.
9. Define retentivity and coercivity.
10. What do you mean by energy product?
11. Mention the energies involved in domains of ferromagnetic materials.
12. Why are ferrites advantageous for use as transformer cores?
13. What are ferrites? And mention its applications.
14. A magnetic field of 2000A/m is applied to a material which has a susceptibility of 1000. Calculate the intensity of magnetization and flux density.
15. Prove that $\mu_r = \mathbf{1+}\ \chi$
16. Define Curie temperature.
17. Define magnetic flux density and magnetic dipole with its unit.
18. What are the four types of energies involved in the growth of magnetic domains?
19. What is meant by reversible and irreversible domains?
20. Define spontaneous magnetization and saturation magnetization.
21. Why diamagnets are called weak magnets and ferromagnetic materials are called strong magnets?
22. Define Curie Weiss law and give its importance.
23. Classify the magnetic materials based on their spin.
24. Is the Cole equation based upon a physical model of tissue?
25. What is a relaxation process in relation to tissue impedance?
26. Does tissue conduct electricity in a similar way to electrons in a metal?

27 Does tissue have both resistive and capacitive properties?

28. What does the parameter α determine in a Cole equation?
29. Is tissue permittivity dominant at very high or very low frequencies?
30. How would you expect the impedance of tissue to change as the measurement frequency is measured?

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Part B Questions

1. Explain Domain theory of ferromagnetism.
2. Differentiate between dia, para and ferromagnetic materials.
3. Explain domain theory of ferromagnetism and the types of energy involved in the process of domain growth in detail.
4. Explain Hysteresis on the basis of domain theory.
5. Differentiate between Hard and Soft magnetic materials.
6. Discuss Weiss theory of ferromagnetism.
7. Explain Cole Cole model in dielectrics.
8. Explain the behaviour of the conductivity and permittivity in dielectrics.
9. Describe in detail the Debye model of relaxation processes.
10. Describe in detail the Cole-Cole model of relaxation processes.

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UNIT 4

WAVES AND OPTICS

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WAVES AND OPTICS

WAVES AND OPTICS

INTRODUCTION

We are all familiar with the idea of a wave; thus, when a stone is dropped into a pond, water waves travel radially outwards; when a violin is played by bow, the strings vibrate and sound waves spread through the hall; when a cell phone tower is transmitting, electromagnetic waves propagate through the space are some of the examples of wave motion. This wave has two inherent physical property during propagation, one energy is propagated through long distance and two disturbance travels through the medium without much permanent displacement in the space. Thus, the ripples spread outwards over a pond carrying energy with them, but as we can see by watching the motion of a small floating body, the water of the pond itself does not move in the direction of propagation of the waves. In this chapter we shall see that whatever be the nature of the medium which transmits the waves, whether it be a stretched string, an electric cable, liquid, air or some other medium, these two properties are common to all these types of wave motion. We can relate this two-property using differential equation called wave equation or, some time named as equation of wave motion, the mathematical part of the wave equation explains the physics of the wave motion by solving the equation with right boundary condition and interpreting the solution suitably.

OSCILLATORY MOTION

There are three main types of simple harmonic motion:

(a) free oscillations – simple harmonic motion with a constant amplitude and period and no external influences

(b) damped oscillations – simple harmonic motion but with a decreasing amplitude and varying period due to external or internal damping forces

(c) forced oscillations – simple harmonic motion but driven externally

Simple harmonic motion is the name given the most uniform of all periodic rectilinear motions. Except perhaps for the propagation of light and other emanations from stars, all motion in some sense is periodic, from the oscillation of atoms, to the swinging of pendulums, to the turning of the Earth on its axis, to the revolution of the planets about the sun, to the revolution of the sun about the Milky way.

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A body that executes simple harmonic motion does so according to the equation

$$x = A\sin\omega t \qquad \text{.....(4.1)}$$

where *x* is the displacement of the body from its equilibrium position at *x = 0*, *t* is the lapse of time and is measured such that at *t = 0*, *x = 0*; *A*, the amplitude of the motion, is the maximum displacement of the body from it equilibrium position, and ω is the angular velocity of the motion, though no geometric angle is involved in the oscillation. The angular velocity ω is related to the period of the oscillation by the equation:

$$\omega = \frac{2\pi}{T} \qquad \text{.....(4.2)}$$

If a body hangs from a spring, stretching the spring because of its weight, a slight displacement of the body from its equilibrium position produces an oscillation. If the spring is again "simple", the oscillation is harmonic. If an additional weight is added to this simple spring, the additional stretch of the spring to a new equilibrium position caused by the weight is proportional the weight added. That is, if Δx is the additional elongation of the spring and ΔW is the additional weight that caused this elongation, then

$$\Delta W = k\Delta x \qquad \text{.....(4.3)}$$

where *k* is the constant of proportionality, the *spring constant*, characteristic of the spring but independent of the elongation or of the weight added, provided that is not stretched beyond its elastic limit.

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There is a simple relationship between the spring constant of a simple spring and the simple harmonic motion it produces. The relationship is that

$$k = m\omega^2 \qquad \text{.....(4.4)}$$

where *m* is the mass of the body attached to the spring.

The body which hangs from a spring is an easily prepared example of oscillatory motion because, except for the pull of gravity and a little friction from the air, only the spring acts on the body. The weight of the body, except for providing the initial elongation of the spring which sets the equilibrium, does not affect the oscillation of the body. However, the mass of the spring, since the spring takes part in the oscillation, must be included in Eq. (4), relating spring and angular frequency. Not all parts of the spring oscillate with the same amplitude, i.e. one end of the spring is fixed and the other end moves with the body. The effect of this is that only one third of the mass of the spring is included in the mass of the oscillating body.

A mass-spring system provides a good illustration of conservation of energy. The kinetic energy of the system is given by

$$K = \frac{1}{2}mv^2 \tag{4.5}$$

.The potential energy is

$$U = \frac{1}{2}kx^2 + mgx \tag{4.6}$$

,

where *x* is the distance away from equilibrium. Neglecting friction, the total mechanical energy

$$K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mgx \tag{4.7}$$

is a constant.

Damped Oscillations

These are oscillations where energy is taken from the system due to friction and so the amplitude decays.

The friction is proportional to the velocity of the object. This includes, air drag (at low velocities), viscous drag and magnetic drag. So, the damped oscillations are of two types:

(i)Natural damping, examples of which are:

internal forces in a spring, fluids exerting a viscous drag.

(ii) Artificial damping, examples of which are:

electromagnetic damping in galvanometers, the coating of panels in cars to reduce vibrations, shock absorbers in cars, interference damping - gun mountings on ships.

The example we have chosen to discuss about the damped harmonic oscillator is a mass suspended in spring connected with and damper immersed in a viscous solution.

To formulate the equation of motion governing the behavior of the oscillation.

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The frictional force acting on the system is of two forms, one Due to the spring force $F_s = -ky$ (k is the spring constant, x is the displacement), second linear damping force due to liquid, the frictional force points in the opposite direction to the velocity $F_f = -\gamma v$ as shown in the fig (). Where ⍰ is the damping coefficient.

The total force on the mass is then $F = -kx - \gamma v$ Therefore the equation according to

$$F_N = ma = m\frac{d^2y}{dt^2} = F_t$$

Newtons

……(4.8)

second law of motion becomes

$$(or) \quad m\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + ky$$

…..(4.9)

To solve the equation, we consider the arbitrary equation $y = Ae^{\alpha t}$ *and* Dividing by m we get and quadratic equation.

$$\alpha^2 + \frac{\gamma}{m}\alpha + \frac{k}{m} = 0$$

…….(4.10)

Where

$$\alpha = -\frac{\gamma}{2m} \pm \sqrt{\frac{\gamma^2}{4m^2} - \frac{k}{m}} = -c \pm \sqrt{c^2 - \omega_0^2} = -c \pm \sqrt{\omega_D^2} \quad \therefore \omega_D\ is\ the\ damping\ frequency$$

The physical nature of the oscillation can be correlated with the equation of motion with three different condetion.

$i)$ $\omega_D^2 < 0$; Underdamped (or) $\gamma/2 < \omega_0$

$ii)$ $\omega_D^2 > 0$; Overdamped (or) $\gamma/2 > \omega_0$

$iii)$ $\omega_D^2 = 0$; Critical damping $\gamma/2 = \omega_0$

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Case 1: when $\gamma/2 < \omega_0$ for small damping or under damped.

$$y = A_1 e^{(-c+i\,\omega_D)t} + A_2 e^{(-c-i\,\omega_D)t} \quad (or)\ \ y = e^{-ct}\left(A_1 e^{i\,\omega_D t} + A_2 e^{-i\,\omega_D t}\right)$$

……(4.11)

On applying the initial condition $\begin{cases} y(t) = \{y_m, & t = 0 \\ y'(t) = \{0, & t = 0 \end{cases}$ to the solution of equation of motion.

$i)\ y(t) = \{y_m, \quad t = 0$

$$y_m = A_1 e^{(-c+i\,\omega_D)\,0} + A_2 e^{(-c-i\,\omega_D)\,0} = A_1 + A_2$$

……(4.12)

$ii)\ y'(t) = \{0, \quad t = 0$

$$0 = (-c + i\,\omega_D)A_1 e^{(-c+i\,\omega_D)\,0} + (-c - i\,\omega_D)A_2 e^{(-c-i\,\omega_D)\,0}$$

$$0 = (-c + i\,\omega_D)A_1 + (-c - i\,\omega_D)A_2 = -c(A_1 + A_2) + i\,\omega_D(A_1 - A_2)$$

$$A_1 - A_2 = -i\frac{c}{\omega_D}y_m$$

……(4.13)

By solving simultaneous equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} y_m \\ -i\dfrac{c}{\omega_D}y_m \end{pmatrix}$$

……(4.14)

Representation in complex form

$$A_1 = \frac{y_m}{2}\left(1 - i\frac{c}{\omega_D}\right) = \frac{y_m}{2}\frac{\sqrt{\omega_D^2 + c}}{\omega_D}e^{-i\frac{c}{\omega_D}}$$

$$A_2 = \frac{y_m}{2}\left(1 + i\frac{c}{\omega_D}\right) = \frac{y_m}{2}\frac{\sqrt{\omega_D^2 + c}}{\omega_D}e^{i\frac{c}{\omega_D}}$$

$\theta = \frac{c}{\omega_D}$ phase lag

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With the above solution the displacement equation becomes

$$y = e^{-ct}\left(\frac{y_m}{\;}\frac{\sqrt{\omega_D^2+c}}{\;}e^{-i\theta}e^{i\,\omega_D t}+\frac{y_m}{\;}\frac{\sqrt{\omega_D^2+c}}{\;}e^{i\theta}e^{-i\,\omega_D t}\right)$$

……(4.15)

$$(or)\quad y=\frac{y_m}{2}\frac{\sqrt{\omega_D^2+c}}{\omega_D}e^{-ct}\left(e^{i\,(\omega_D t-\theta)}+e^{-i\,(\omega_D t-\theta)}\right)$$

$$(or)\ \boldsymbol{y(t)} = \boldsymbol{Y_m e^{-ct}\cos(\omega_D t-\theta)}$$

……(5.16)

$\therefore$ *damping amplitude* $\boldsymbol{Y_m} = y_m\frac{\sqrt{\omega_D^2+c}}{\omega_D}$; $\theta=\frac{c}{\omega_D}$ phase lag due to the damping velocity of the liquid.

The under damping of oscillator is a combination of exponential term with damping coefficient and simple harmonic *cos* function.

Case ii) over damping $\gamma/2 > \omega_0$

If we makes the resistive force even stronger, the **over-damped** oscillator decays in an even quicker manner. For the $\gamma/2 > \omega_0$ argument of the square root becomes negative,

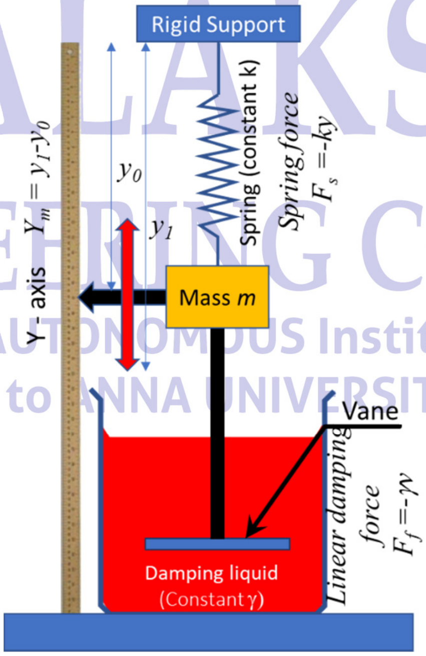

Fig (4.1) Setup for damped harmonic oscillation.

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Case iii) critical damping $\gamma/2 = \omega_0$

The frequency ω_D vanishes and the expression in the exponent reduces to

$$\alpha = -\frac{\gamma}{2m} \because \ \omega_D = 0$$

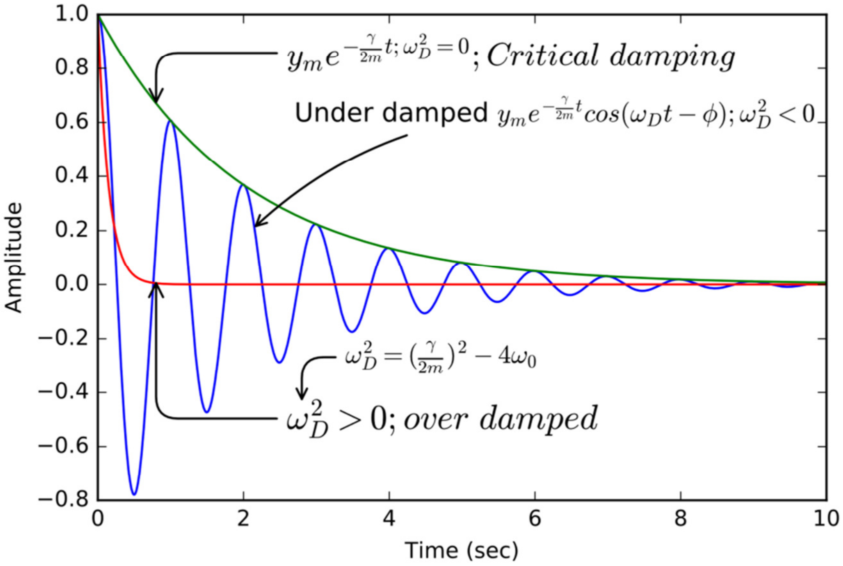
The solution no longer has an oscillatory part. In addition, we no longer have two solutions

that can be used to fit arbitrary initial conditions.

The general solution as a function of time becomes

$$y = e^{-\frac{\gamma}{2m}t}(A_1 + A_2) = y_m\, e^{-\frac{\gamma}{2m}t} \qquad \text{……(4.17)}$$

then the oscillator never really oscillates: it decays exponentially, just barely not reaching the other side of the equilibrium.



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Fig(4.2). Damped oscillation with all three conditions.

4.2 Forced Harmonic Oscillation

We saw earlier, in Section (2.2.1), that if a damped mechanical oscillator is set into motion then the oscillations eventually die away due to frictional energy losses. In fact, the only way of maintaining the amplitude of a damped oscillator is to continuously feed energy into the system in such a manner as to offset the frictional losses. A steady (i.e., constant amplitude) oscillation of this type is called *driven damped harmonic oscillation*.

Let's drive our damped spring object system by a sinusoidal force. Suppose that the y-component of the driving force is given by

$$F_y = F_0 cos(\omega t)$$

……(4.18)

Where $F_0$is called the amplitude (maximum value) and $\omega$is the driving angular frequency. The force varied between F_0 and $-F_0$ because the cosine function varies between +1 and -1. Define y(t) to be the position of the object with resect to the equilibrium position. The y- component of the force acting on the object is now the sum

$$F_y = F_0\, cos(\omega t) - ky - \gamma \frac{dy}{dx}$$

……(4.19)

Newton's Second law in the y-direction becomes

$$\frac{d^2y}{dt^2} + \frac{\gamma}{m}\frac{dy}{dt} + \frac{k}{m}y$$

……(4.20)

We can rewrite Eq. (2.20) as

$$\frac{d^2y}{dt^2} + B\frac{dy}{dt} + \omega_0 y$$

……(4.21)

we begin by assuming a solution of the form

$$y(t) = A\cos(\omega t$$

……(4.22)

where the amplitude A and phase constant θ need to be determined. We begin by defining the complex function

……(4.23)

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$$z(t)$$

our desired solution can be found by taking the real projection.

$$y(t) = Re(z(t))$$

differential equation of motion can be written as

$$\frac{d^2z}{dt^2} + B\frac{dz}{dt} + \omega_0 z$$

……(4.24)

$$\boldsymbol{\frac{dz}{dx}} = \boldsymbol{i\omega A}e^{i(\omega t+\theta)} = i\omega z$$

$$\boldsymbol{\frac{d^2z}{dt^2}} = \boldsymbol{-\omega^2 A}e^{i(\omega t+\theta)} = \boldsymbol{-\omega^2} z$$

we substitute the above equation into equation of motion we get.

$$\frac{F_0}{m}e^{i\omega t} = (= \boldsymbol{-\omega^2 m} + \boldsymbol{B}i\omega + \omega_0{}^2)z = (\omega_0{}^2 - \boldsymbol{\omega^2 m} + \boldsymbol{B}i\omega)\boldsymbol{A}e^{i(\omega t+\theta)}$$

……(4.25)

Or

$$\boldsymbol{A}e^{i\theta} = \frac{F_0/m}{(\omega_0^2-\omega^2)+iB\omega} = \frac{F_0}{m}\left[\frac{(\omega_0^2-\omega^2)-iB\omega}{(\omega_0^2-\omega^2)^2+B^2\omega^2}\right]$$

$$\boldsymbol{A} = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+B^2\omega^2}} = \frac{F_0/m}{\sqrt{\left(\omega_0^2-\frac{k}{m}\right)^2-\frac{\gamma^2}{m^2}\omega^2}}$$

……(4.26)

And $\theta = tan^{-1}\left[\frac{(\gamma/m)\omega}{(\omega^2-\omega_0^2)}\right]$

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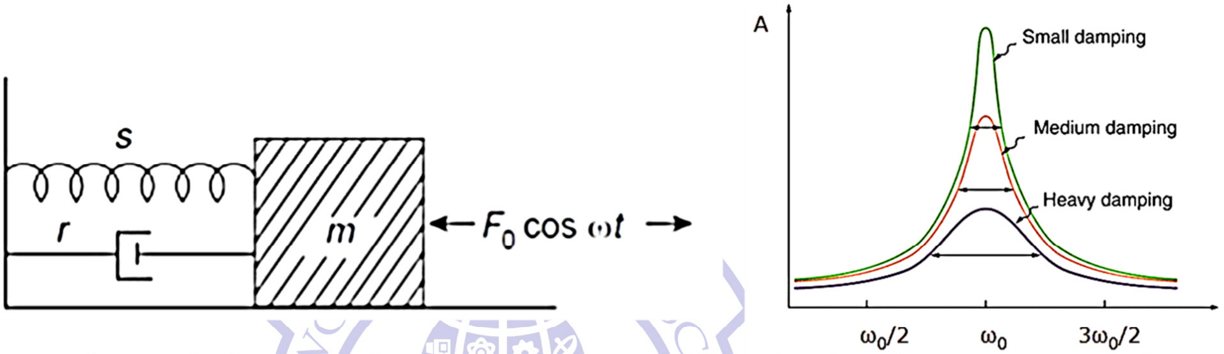
Resonance

When $\frac{\gamma}{m} \ll 2\omega_0$ we say that the oscillation is lightly damped. For a lightly-damped driven oscillator, after a transitory period, the position of the object will oscillate with the same angular frequency as the driving force. The plot of amplitude A vs driving angular frequency w for a lightly damped force oscillator is show in fig(2.3). If the angular frequency is increased from zero, the amplitude of the A will increase until it reaches a maximum when the angular frequency of the driving force is the same as the natural angular frequency. Associated with the undamped oscillator. This is called resonance. When the driving angular frequency is increased above the natural angular frequency the amplitude of the portion oscillations diminishes.



a) Setup for forced oscillation b) Amplitude vs frequency to study damping

Fig. 4.3 – Set-up for Forced Oscillation

4.3 Plane Progressive Wave:

A wave propagating from one point to another in a medium without being subjected to any boundary condition, is called a progressive wave.

Characteristics of progressive wave

1) Every particle describes simple harmonic motion along the direction of propagation of wave, there being a change of phase from point to point.
2) The wave velocity in a given medium is a constant determined by the density and the elastic constant of the medium.
3) Only the energy is carried by the advancement of the waveform in the direction of propagation of the wave.
4) The phase difference between two vibrating particles on the line of propagation is proportional to the path difference between the particles.

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Equation of plane progressive wave

When an wave moves along the positive direction of x with a velocity v. let the displacement at any instant of time t at x=0 is

$$y = a \sin \omega t$$

……(4.27)

Here, v be the wave velocity. We have for λ displacement phase change is 2π.

So for x displacement the phase change $\phi = \frac{2\pi}{\lambda}x$

So we get motion at P is

$$y = a\sin(\omega t - \phi)$$

……(4.28)

$$y = a\sin(\omega t - kx) \; where \; k = \frac{2\pi}{\lambda} \; propagation \; constant$$

$$y = a\sin\left(\omega t - \frac{2\pi}{\lambda}x\right) = f(vt - x)$$

……(4.29)

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

……(4.30)

Where $\omega = \frac{2\pi v}{\lambda}$

If the wave moves towards the negative direction of the x-axis, the displacement

$$y = a\sin\frac{2\pi}{\lambda}(vt + x) = f(vt + x)$$

……(4.31)

4.4 Phase velocity

At any instant in a progressive wave, the quantity $vt - x$ and hence $f(vt - x)$ remains the same at all points on a plane perpendicular to the x-axis. Thus the wave fronts are plane., so that $f(vt - x)$represents a plane wave propagating in the positive x-direction. The quantity $vt - x$ is the phase.

Let $y(x,t)$ denotes the value of the field parameter at x at time t. As the wave moves in the positive x-direction, the same value occurs at $x + dx$ at time $t + dt$.

Hence

$$y(x,t) = y(x + dx, t + dt) = constant$$

$$f(vt - x) = f(v(t + dt) - (x + dx)) = constant$$

$$f(\omega t - kx) = f(\omega(t + dt) - k(x + dx)) = constant$$

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So, we get

$$vt - x = constant$$

$$vdt - dx = 0$$

$$v = \frac{dx}{dt} = v_p$$

Similarly, in terms of angular frequency we get.

$$\omega t - kx = constant$$

$$\omega dt - kdx = 0$$

$$v = \frac{dx}{dt} = \frac{\omega}{k} = v_p$$

……(4.32)

Hence, wave velocity $v = v_p$ phase velocity

4.5 Group velocity.

If two or more plane simple harmonic waves of the same amplitude but different frequencies superimpose, a group of waves are formed. The amplitude of the group changes with distance, and the velocity with which the maximum of the wave group travels is referred to as the group velocity. The energy is transmitted with the group velocity.

We consider two waves of equal amplitude A and slightly different angular frequency $\omega \text{ and } \omega + d\omega$, traveling with the propagation constant $k \text{ and } k + dk$, i.e. the displacement

$$y_1 = a\sin(\omega t - kx)$$

……(4.33)

And

$$y_2 = a\sin\big((\omega + d\omega)t - (k + dk)x\big)$$

……(4.34)

So, the resultant displacement

$$y = y_1 + y_2 = a\sin(\omega t - kx) + a\sin\big((\omega + d\omega)t - (k + dk)x\big)$$

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$$y = 2a\cos\left(\frac{td\omega - \ xdk}{2}\right)\sin\left(\left(\omega + \frac{d\omega}{2}\right)t - \left(k + \frac{dk}{2}\right)x\right)$$

……(4.35)

The phase velocity of the composite wave is

$$v_p = \frac{\omega + d\omega/2}{k + dk/2} \neq \frac{\omega}{k}$$

……(4.36)

The amplitude represented by the cosine term advance with the group velocity v_g. If the phase $\frac{td\omega - xdk}{2}$ associated with it progresses to $(x + dx) at\ time\ (t + dt)$, we have

$$td\omega - \ xdk = (t + dt)d\omega - (x + dx)dk$$

So, the group velocity

$$v_g = \frac{dx}{dt} = \frac{d\omega}{dk}$$

Again, we have

$$\omega = vk$$

$$d\omega = vdk + kdv$$

$$v_g = v_p + k\frac{dv}{dk}$$

……(4.37)

2.2.6 Wave equation

If $f(vt \pm \ x)$ is the solution for harmonic wave equation.

We can write $f(vt \pm \ x) = f(u), where\ u = vt \pm \ x$

So $\frac{\partial u}{\partial x} = \pm 1\ and\ \frac{\partial u}{\partial t} = \ vt$

Now, use the chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} \quad and \quad \frac{\partial f}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial t}$$

……(4.38)

So

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \Rightarrow \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial u^2} \; and \; \frac{\partial f}{\partial t} = v\frac{\partial f}{\partial u} \Rightarrow \frac{\partial^2 f}{\partial t^2} = v^2\frac{\partial^2 f}{\partial u^2}$$

……(4.39)

On re arranging the terms we get.

$$\frac{\partial^2 f}{\partial u^2} = \frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

……(4.40)

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

……(4.41)

The above equation is called the 1D wave equation.

Physics of Light

Characteristics of light

Light is an **electromagnetic wave**. As shown below, an **electromagnetic wave** is a transverse wave consisting of mutually perpendicular oscillating electric and magnetic fields. Electromagnetic waves are ultimately produced by an accelerating charge. A changing electric field produces a changing magnetic field which in turn produces a changing electric field and so on. Because of this relationship between the changing electric and magnetic fields, an **electromagnetic wave is a self-propagating wave that can travel through a vacuum or a material medium since electric and magnetic fields can exist in either one.**

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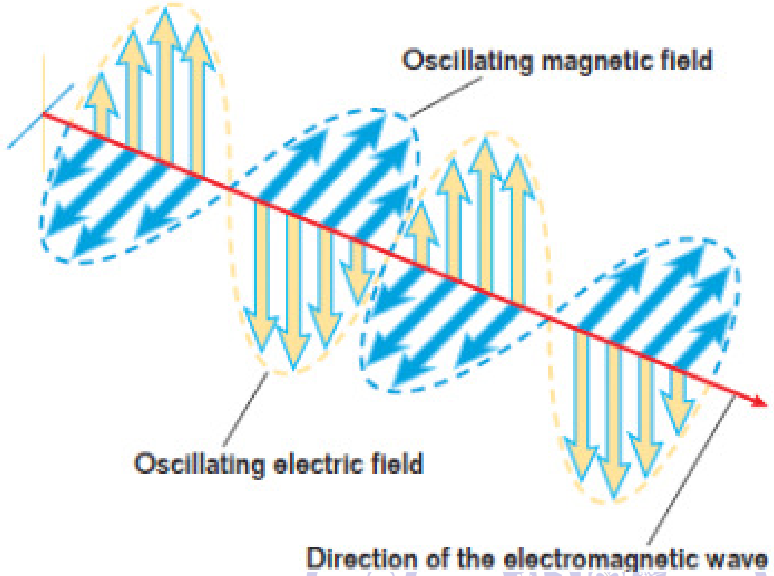

All **electromagnetic waves travel at the speed of light**. Electromagnetic waves are distinguished from each other by their differences in wavelengths and frequencies (wavelength is inversely related to frequency).

$c = f\lambda$ c = speed of light = 3.0 x 10 8 m/s in a vacuum = 300,000 km/s = 186,000 miles/s

A light year is the distance that light travels in one year in a vacuum

The electromagnetic spectrum is a continuum of all electromagnetic waves arranged according to frequency and wavelength. Visible light is only a small portion of the entire electromagnetic spectrum.

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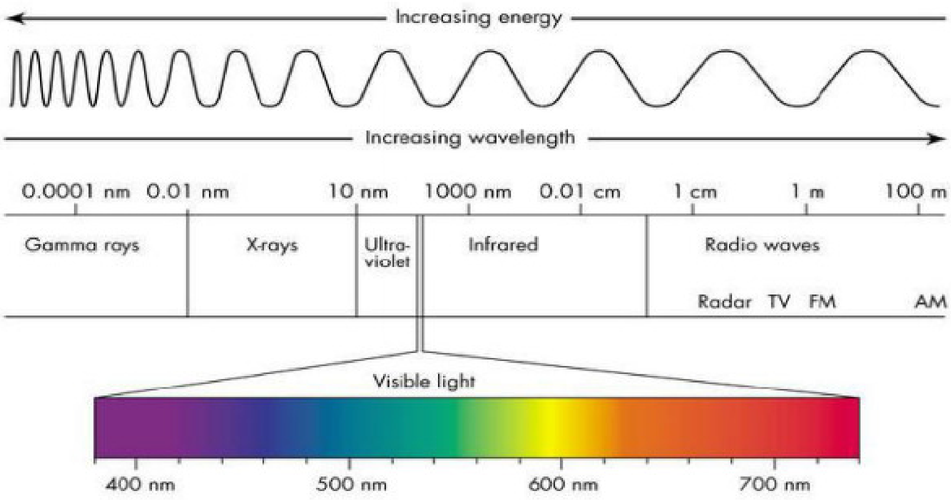

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Color

- The color of an object depends on:

1. which wavelengths of light shine on an object,

2. which wavelengths are absorbed,

3. and which are reflected.

- If all wavelengths of incoming light are completely reflected by an object, that object appears the same color as the light illuminating it. An object of a particular color absorbs light of all colors except light whose color is the same as the object's color. As shown to the right, a leaf appears green under white light because the pigment in the leaf reflects only green light.

- Sunlight is composed of all colors of visible light. We call sunlight "white light" because when all wavelengths of light are present they produce white. White is not a color - it is the combination of all colors. Black is not a color either…it is the absence of light. An object looks black when it is absorbing all the light that hits it. When objects absorb light, they become warmer due to the absorption of energy.

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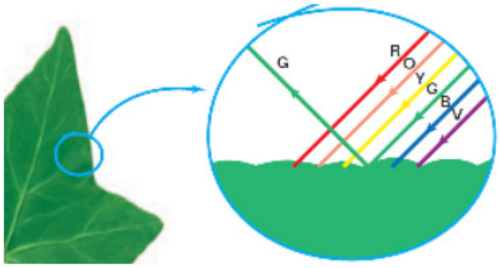
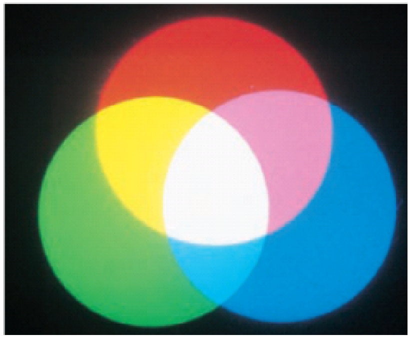
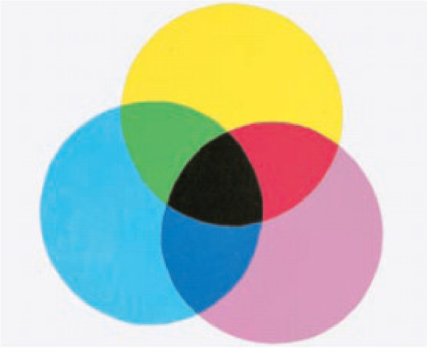


Color addition produces colors by adding colors of light. The three additive primary colors of light are red, blue, and green and when they are mixed with equal intensity they produce white. The additive primary colors are used in color monitors and TV's to produce all the colors



Color subtraction produces different colors by mixing pigments. Pigments absorb or subtract light of certain wavelengths and reflect others. This is used in magazines, paints, and plastics. The primary colors of pigment (light subtraction) are magenta, cyan, and yellow. If you mix these together in the right amounts, you will get a pigment that will absorb all light and appear black



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Color phenomenon in nature Atoms and molecules in substances vibrate and can resonate when light hits and interacts with them. This is what causes some wavelengths or frequencies to be absorbed by certain objects and determines which of the "three fates" will come to light that encounters matter. If the light encounters solids or liquids and the light matches the natural frequency of the substance, the atoms will resonate, absorb the light and turn it into heat in the substance. In low-density gases, this resonance does not result in complete absorption and conversion to heat. This concept explains several natural phenomena:

- **Why is the sky blue? 3 important factors explain this**

1. Molecules in the atmosphere are very small molecules of nitrogen and oxygen and they resonate in the frequencies of violet and blue visible light.

2. Since air or gas molecules are so far apart, their resonance does not cause them to absorb the light. Instead the vibrations just scatter out the violets and blues in all directions…including down to the surface and to your eyes.

3. Violet is actually scattered out the most, but our eyes are not very sensitive to violet…Consequently we see a mostly blue sky!

- **Why are sunsets red? 2 important factors in this:**

1. The sun's light must travel a much longer path through the atmosphere to get to you at sunset and all the atmosphere keeps scattering out more of the high frequency violets and blues as the light travels through.

2. Once all the higher frequencies are scattered out, then all that is left to go straight through to you is the lower frequencies like the orange and reds. So you see an orange - red sunset!

- **Consider this - if our atmosphere was made up of molecules that resonate in long wavelengths of light, what color would our midday sky be? What would be the color of our sunsets and sunrises?**

- **Why is water greenish-blue (cyan)?**

1. Liquids produce a different effect than gases when they resonate with certain colors/frequencies of light. Water molecules resonate with infrared and low frequency visible light (reds and oranges). When molecules in solids or liquids resonate with light, it results in those resonant wavelengths or colors being absorbed and/or scattered…after some depth through water all that is left to reach a receiver (such as your eyes) is the higher frequency blues and greens. Blue and green combine to make cyan.

   If you were to take a red object deep under water - it would appear black because no red light would reach the object to be reflected to your eyes.

Physics of Sound

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Sound is produced in a material medium by a vibrating source. Sound is a longitudinal, mechanical wave.Sound can travel through any medium, but it cannot travel through a vacuum. There is no sound in outer space.

Sound is a variation in pressure. A region of increased pressure on a sound wave is called a compression (or condensation). A region of decreased pressure on a sound wave is called a rarefaction (or dilation).

Apart from the sound waves that can be heard by human ear [Between 20 Hertz to 20,000 Hertz] there are sound waves of frequencies less than 20 Hertz called infrasonic and sound waves greater than 20,000 Hertz called Ultrasonic

Ultrasonic Production

There are three methods for producing Ultrasonic waves. They are:

(i) Mechanical generator or Galton's whistle.

(ii) Magnetostriction generator.

(iii) Piezo-electric generator

Magnetostriction method

Principle:

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When a magnetic field is applied parallel to the length of a ferromagnetic rod made of material such as iron or nickel, a small elongation or contraction occurs in its length. This is known as magnetostriction. The change in length depends on the intensity of the applied magnetic field and nature of the ferromagnetic material. The change in length is independent of the direction of the field.

When the rod is placed inside a magnetic coil carrying alternating current, the rod suffers a change in length for each half cycle of alternating current. That is, the rod vibrates with a frequency twice that of the frequency of A.C. The amplitude of vibration is usually small, but if the frequency of the A.C. coincides with the natural frequency of the rod, the amplitude of vibration increases due to resonance.

When a ferromagnetic rod is subjected to a magnetic field parrell to its lkength it undergoes a change in length.

The magnetostrictive oscillator is based on magetostriction effect. Instead of a d.c magntic field if an magnetic field if an a.c magnetic field is applied to the ferromagnetic rod itv expands anmd contracts alternately. If the frequency of the applied field becomes equal to the naturalk vibrating frequency of the rod large amplitude of mechanical vibratins are produced in the rod. This resuls in the generation of ultrasonic waves and are emitted from the ends of thew rod.the frequency of vibration of the rod is

$$n= 1/2L \sqrt{E / \rho}$$

Where Ĺ – length of the rod

E - youngs modulus of the rod

ρ - density of the rod.

Construction

AB is a permanently nickel rod which is clamped in the middle between two knife edges. Coil L is wound on the right hand portion of the rod along with a vriable capacitor C coil L and the capacitor forms the resonant circuit. Coil l wound on the left hand portion of the rod is connected in the base circuit. The coil l is used as a feed back loop.

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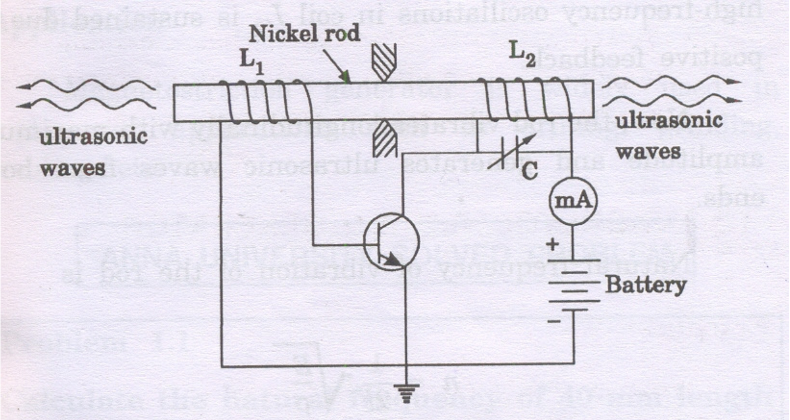

Fig. 1.2. Magnetostriction generator circuit

Working

When E is switched on the circuit l c in the collector circuit of transistor sets up an alternating current of frequency. This alternating current flowing round the coil l produces an alternating magnetic field of frequency f along the length of the rod AB. Then the rod starts vibrating due to magnetostrictive effect. The vibrations of the rod create ultrasonics which are sent out. The coil l helps in increasing the amplitude of ultrasonic waves. The longitudinal expansion and contraction of the rod AB produces an e.m.f in the coil l which is applied to the base of the transistor which increases the amplitude of high frequency oscillations in coil l due to positive feed back. By adjusting the capacitor c the frequency of the a.c current can be made equal to the natural vibrating frequency of the rod. The resonance condition is indicated by the rise in the collector current shown on the milliammeter.

Advantages

1. The ac circuit is simple and low cost.

2. High power output is possible without damaging the circuit.

Disadvantages

1. The output frequency is limited (upto 300 khz only)
2. The output frequency greatly depends on temperature.

Piezoelectric effect:

When crystals like quartz or tourmaline are stressed along any pair of opposite faces, electric charges of opposite polarity are induced in the opposite faces perpendicular to the stress. This is known as Piezoelectric effect.

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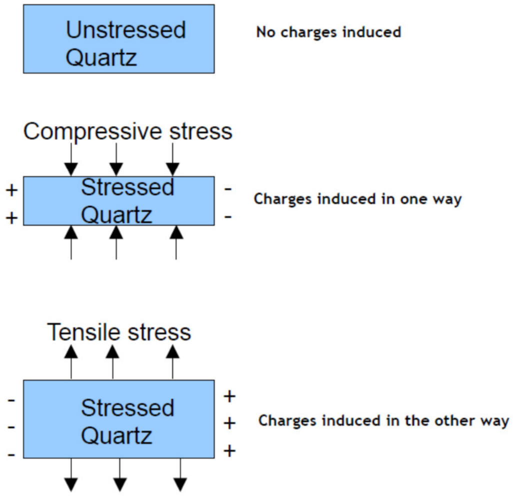

Piezoelectric effect- Mechanism:

Piezoelectric and inverse piezoelectric effects are only exhibited by certain crystals which lack centre of symmetry. In a piezoelectric crystal, the positive and negative electrical charges are separated, but symmetrically distributed, so that the crystal overall is electrically neutral. Each of these sides forms an electric dipole and dipoles near each other tend to be aligned in regions called Weiss domains. The domains are usually randomly oriented, but can be aligned during *poling* , a process by which a strong electric field is applied across the material, usually at elevated temperatures. When a mechanical stress is applied, this symmetry is disturbed, and the charge asymmetry generates a voltage across the material.

where the application of an electrical field creates mechanical deformation in the crystal.

Inverse piezoelectric effect:

When an alternating e.m.f is applied to the opposite faces of a quartz or tourmaline crystal it undergoes contraction and expansion alternatively in the perpendicular direction. This is known as inverse piezoelectric effect. This is made use of in the piezoelectric generator.

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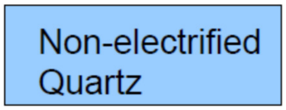
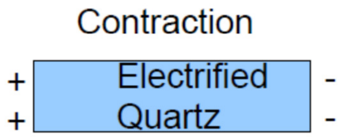
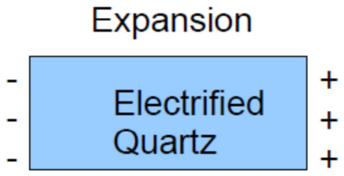


It experiences neither contraction nor expansion



It experiences contraction equivalent to compressive forces acting on the faces normal to the charge induced faces



It experiences expansion equivalent to tensile forces acting on the faces normal to the charge induced faces

Piezoelectric method

When an alternating voltage is applied to one pair of opposite faces of a quartz crystal, alternative mechanical contractions and expansions are produced on the other pair of opposite faces of the crystal. The crystal is thus, set into mechanical vibrations.

When the frequency of the applied AC voltage is set equal to the natural frequency of the crystal, then resonance will occur. Vibrations of larger amplitude of sound are produced.

If the frequency of the AC voltage is in the ultrasonic range, the crystal would produce ultrasonic waves.

Construction

The circuit diagram of tuned base oscillator circuit. A slice of quartz crystal is placed between the two metal plates A and B. The coil L_1 of oscillator circuit is taken from primary of transformer and coil L_2 from secondary of the transformer.

The plates A and B are coupled to the electronic oscillator through secondary (L_3)

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Of the transformer. The coil L_2 and variable capacitor C from – tank circuit of the oscillator.

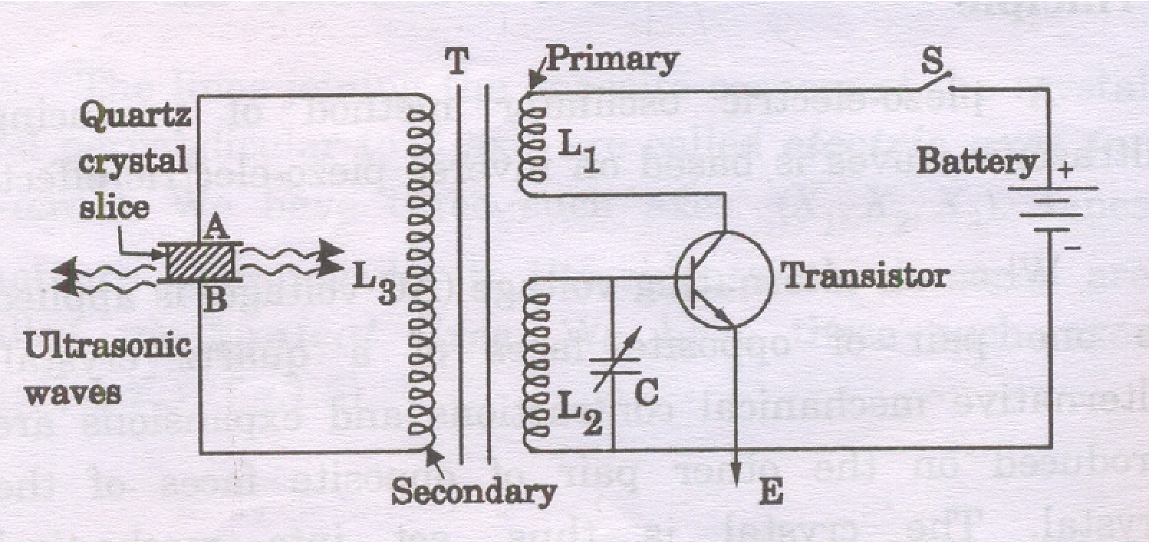

Fig. 1.6. Piezo electric oscillator

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Working

When the battery is switched on, the electronic oscillator produces alternating voltages with frequency

$$f = 1 / 2\pi\sqrt{L_2C}$$

An oscillatory a.m.f is induced in the coil L_3 due to transformer action.

The crystal is now under high-frequency alternating voltage. The crystal expands and contracts alternately due to inverse piezo-electric effect. The crystal is thus set into mechanical vibrations.

The capacitance of the variable condenser C is adjusted such that the frequency of the applied

AC voltage is equal to the natural frequency of the crystal. Now, the crystal vibrates with large amplitude due to resonance. Thus, high-power ultrasonic waves are produced from the crystal.

Suppose, there is an X-cut crystal plate of length L, then the frequency of its length-wise vibration is

$$n = \frac{p}{2l \; \sqrt{E/\rho}}$$

Where p = 1, 2, 3, ….

E is Young’s modulus of the crystal

P is the density of the crystal plate

Merits:

1. More efficient than magnetostriction oscillator.
2. Frequencies as high as 5×10^8 Hz
3. Not affected by temperature and humidity.

Demerit:

1. Cost is very high.
2. Cutting and shaping is very difficult.

Acoustic Grating

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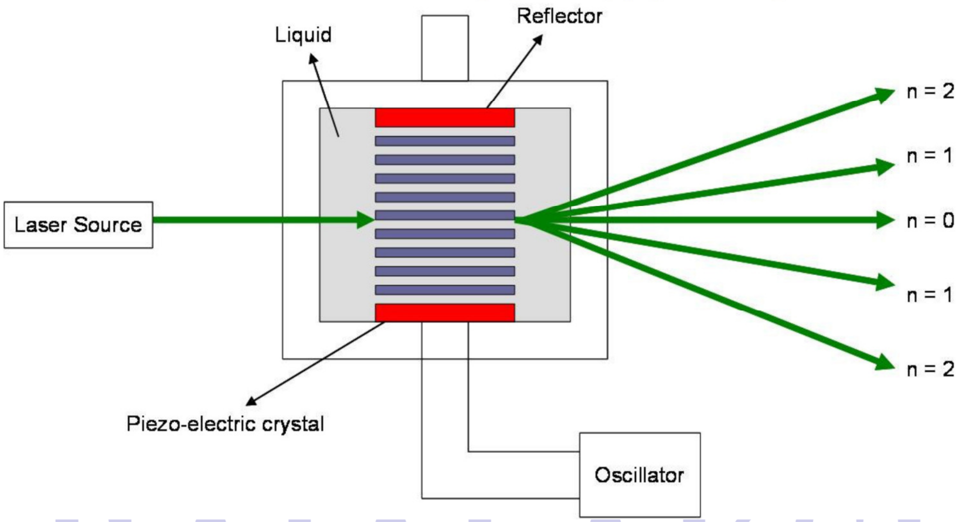
When ultrasonic waves travel through the transparent longitudinal stationary waves are formed due to alternating rarefactions and compressions

If a monochromatic light is passed perpendicular to these waves the liquid medium behaves as a diffraction grating. The regions of compressions act as opaque medium and rarefactions act as transparent medium for light waves such a grating is known as acoustic grating.

Acoustic grating behaves in the same way as ruled diffraction grating.. it is used to find wavelength and velocity of ultrasonic waves in liquid.



Theory

When the light from a monochromatic sources like sodium vapour lamp or laser passes through the acoustic grating then the diffraction grating formula is given by

$$(a+b) \sin \theta_{n\,=} \sin \theta_n = n\lambda \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (1)$$

Where d = (a+b) the distance between two successive noides or antinodes of stationary waves.

Where n is the order of diffraction

λ is the wavelength of light used

θ_n is the angle of diffraction for nth order

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Let λ_m be the wavelength of ultrasonic waves in medium then

$$d = \lambda_m/2 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(2)$$

Substituting equation (2) in equation (1), we have

$$\lambda_m/2 \ \sin\theta_n = n\lambda$$

$$\lambda_m = 2n\lambda / \sin\theta_n \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(3)$$

Let f be the frequency of ultrasonic waves. If v is the velocity of ultrasonic waves in the given liquid, the

$$v = f \ \lambda_m$$

$$v = 2n\lambda \ f/\sin\theta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(4)$$

Experimental arrangement

Acoustical grating formation can be easily demonstrated in the laboratory. In an ultrasonic cell the liquid under study is taken ultrasonic transducer is fixed at one side wall inside the cell and ultrasonic waves are generated the waves traveling from the transducer get reflected from the opposite wall and standing wave pattern called acoustical grating

If light from diode laser is allowed to pass through the liquid in a direction perpendicular to the grating diffraction takes place and on the screen placed at a distance the higher order diffracted images can be seen.The angular separation between the direct ray and diffracted rays of different orders can be calculated by measuring the distance of diffracted beams from the central beam. Using the given formula the wavelength and velocity of ultrasonic waves in liquid can be calculated

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Uses

This method is useful in measuring the velocity of ultrasonic waves through liquids and gases at various temperatures from these measurements many parameters of the liquid as free volume compressibility etc can be calculated.

Nondestructive testing

The ultrasonic pulse echo system is used to detect flaw, evaluate the dimension of the flaw,characterize a material, find hidden cracks, voids, porosity, and other internal discontinuities inmetals, composites, plastics, and ceramics, and so on.

Pulse Echo System

An ultrasonic pulse echo system basically consists of the following several functional units.

1. A master timer (an electronic clock)
2. An electronic (signal) pulse generator
3. Signal transmitting transducer.
4. Reflected signal receiving transducer.
5. An echo signal amplifier and
6. A display device such as CRO

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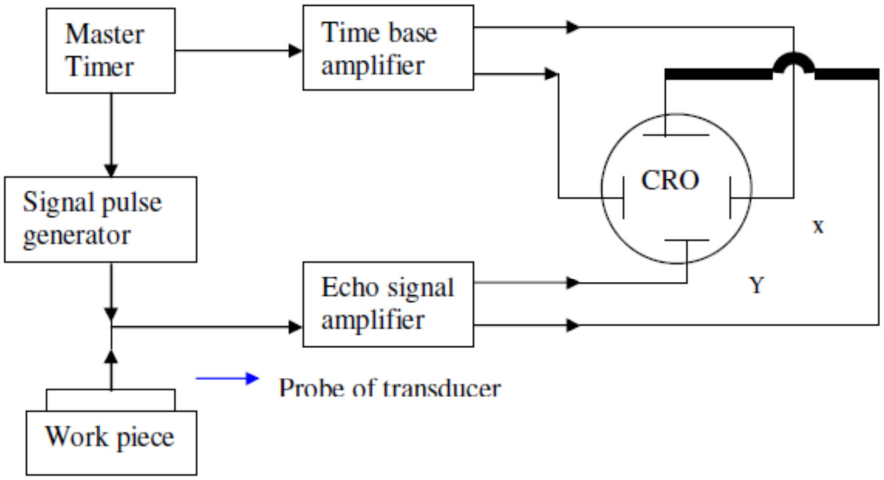

The master timer (the electronic clock) triggers the signal pulse generator at regular intervals. Signal generator sends a short burst of high frequency alternating voltage to the transducer. The transducer generates pulse of ultrasonic waves. This sound energy propagates through the inspection piece. When there is a discontinuity such as crack in the wave path, part of energy is reflected back from

the flaw surface. The reflected pulse causes the receiving transducer element to vibrate inducing alternating electrical voltage across the transducer. This induced voltage is instantaneously amplified and fed to display device (CRO). Initially when the master timer triggers the signal pulse generator, simultaneously it activates linear time base circuit of the display device (CRO). Hence in CRO, the

reflected signal strength versus the time is displayed.

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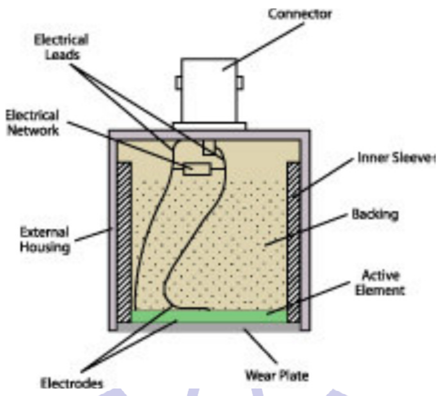
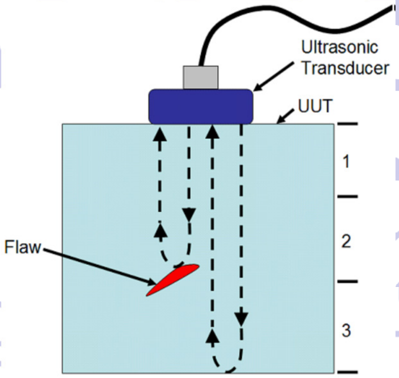
Procedure to find flaw



Cross section of a typical contact transducer

Ultrasonic flaw detection is basically a comparative technique. Using appropriate referencestandards along with knowledge of sound wave propagation and generally accepted test procedures, a trained operator identifies specific echo patterns corresponding to the echo response from good parts and from representative flaws. The echo pattern from a test piece may then be compared to the patterns from these calibration standards to determine its condition.

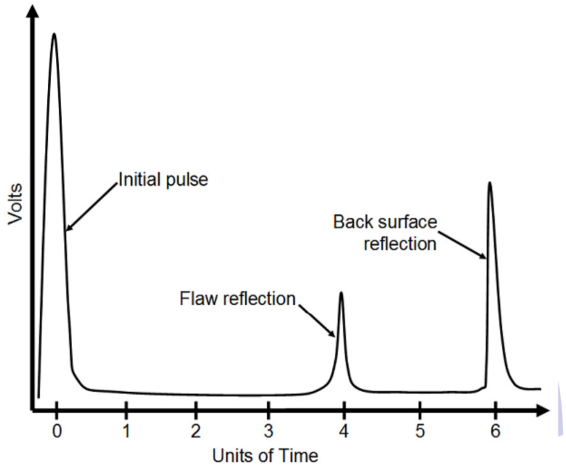
Reflection Mode



Let ‘d’ be the distance of flaw from the transducer; ‘v’ be the velocity of sound in the test specimen and‘t’ be the time of flight. Then the total distance traveled by the sound in the specimen=2d.

□d□=vt/2

The following diagram shows such echo pattern indicating the presence of a defect.



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Merits:

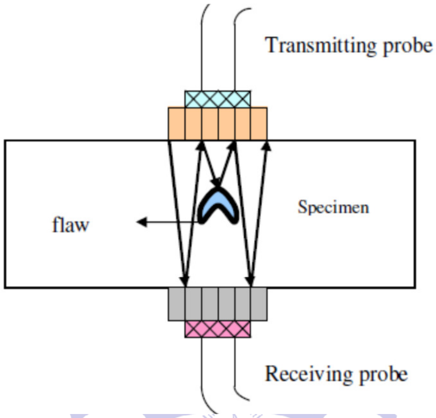
1. Only one side of the specimen is required for the study.

2. The specimen may be of any shape (both the sides of the wall to be studied need not be parallel)

3. Since single probe is used for transmission and reception coupling loss is minimum.

4. The depth of the defect from the surface can be measured

Transimission Mode



In this method the transmitting probe is placed in contact with the test piece surface on one side and a receiving probe is placed exactly on the opposite side of the test piece as shown in the figure. Both the transmitting probe and receiving probe are moved together. When there is no defect, the receiving probe receives full signal and when there is a defect between the transmitter and receiver the amplitude of the received signal is reduced because of partial reflection at the defective region.

Demerits

1. Both the sides of the specimen are required for the study.

2. Both the sides of the wall to be studied need to be parallel.

3. Since two different probes are used for transmission and reception coupling loss is maximum.

4. Only the presence of the defect is revealed. Its depth is not made known.

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SCANNING TECHNIQUES

Any discontinuity in the medium of propagation of an ultrasonic pulse gives rise to back-reflection which travels back to the input and is called the echo of the launched pulse. Ultrasonic pulse-echo data therefore comprises of

(i) time elapsed between the launch of the pulse and the received echo and

(ii) intensity /energy of the received echo. There are different ways in which this data can be presented for visualization/measurement.

The three most common formats are known in the NDT(Non-Destructive

Testing) world as A-scan, B-scan and C-scan presentations.

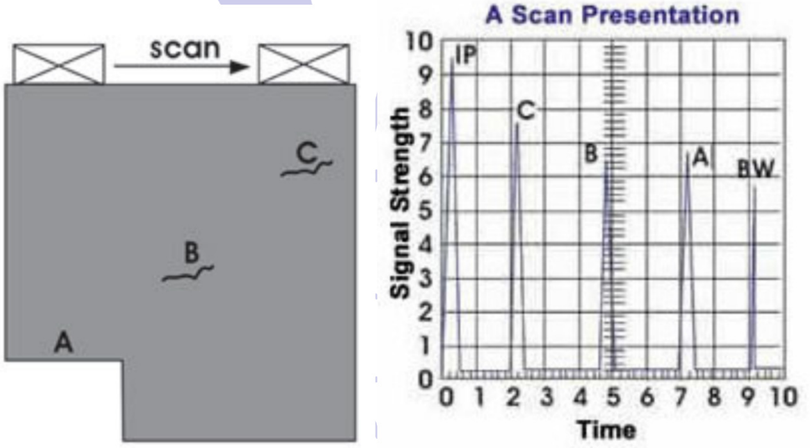
Each presentation mode provides a different way of looking at and evaluating the region of material being inspected. Modern computerized ultrasonic

scanning systems can display data in all three presentation forms simultaneously

The A-mode

The A-mode or the Amplitude mode of data presentation displays the amount of received ultrasonic energy as a function of time. The amount of received energy is plotted along the vertical axis (Y-axis) and the elapsed time (which may be related to the sound energy travel time within the material) is displayed along the horizontal axis (X-axis). In the A-scan presentation, the depth from which the

reflection is obtained can be determined by the position of the signal on the horizontal sweep. Let us look at the sample piece with discontinuities as indicated in the figure 1 given below at B And C. Let the piece have a different depth, A in the initial region when compared to the rest.



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In the illustration of the A-scan presentation to the right, the initial pulse generated by the transducer is represented by the signal IP, which is near time zero. As the transducer is scanned along the surface of the part, four other signals are likely to appear at different times on the screen.

_ When the transducer is in its far left position, only the IP signal and signal A, the sound energy reflecting from surface A, will be seen on the trace. _ As the transducer is scanned to the right, a signal from the backwall BW will appear later in time, showing that the sound has traveled farther to reach this surface.

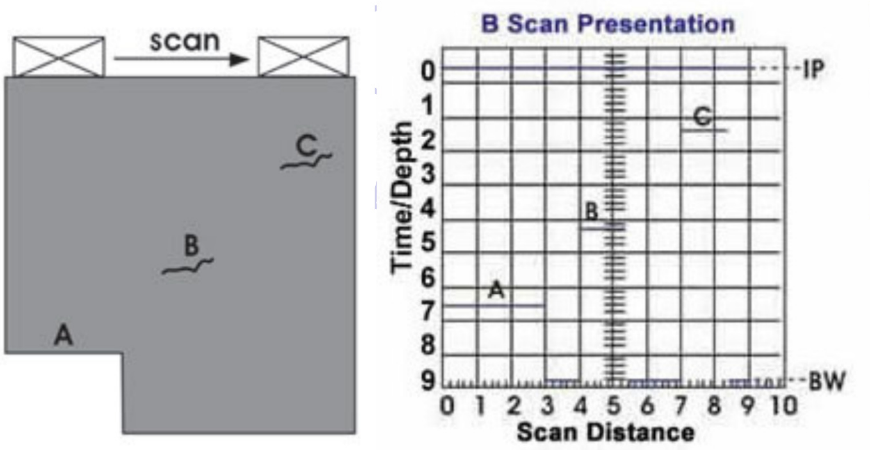
_ When the transducer is over flaw B, signal B will appear at a point on the time scale that is approximately halfway between the IP signal and the BW signal. Since the IP signal corresponds to the front surface of the material, this indicates that flaw B is about halfway between the front and back surfaces of the sample.

When the transducer is moved over flaw C, signal C will appear earlier in time since the sound travel path is shorter and signal B will disappear since sound will no longer be reflecting from it. If the input pulse is of sufficiently high energy and the size of discontinuity in the direction of propagation is small, then along with the signal from B or C , we may be able to observe the echo fromthe backwall BW also.

B-Mode

In the B-mode or Brightness mode of display, the time-of-flight (travel time) of the sound energy is displayed along the vertical axis with zero of the time scale at the top left corner and the linear position of the transducer is displayed along the horizontal axis. This format reveals the depth of the discontinuity and its approximate linear dimensions in the scan direction. The B-scan presentation

is hence like a profile (cross-sectional) view of the test specimen.



The B-scan is typically produced by establishing a trigger gate on the A-scan. Whenever the signal intensity is great enough to trigger the gate, a point with appropriate brightness is

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produced on the Bscan. The gate is triggered by the sound reflecting from the backwall of the specimen and by smaller reflections from the discontinuities within the material. Thus each reflection is represented by a spot

and the intensity or energy of the reflection in represented by the brightness of the spot.In the B-scan image above, line A is produced as the transducer is scanned over the reduced thickness portion of the specimen. When the transducer moves to the right of this section, the backwall line BW is produced. When the transducer is over flaws B and C, lines that are similar to the length of the flaws and at similar

depths within the material are drawn on the B-scan. It should be noted that a limitation to this display technique is that discontinuities may be masked by larger reflections near the surface

C-mode

The C-scan presentation provides a plan-type (2D) view of the location and size of test specimen features. The plane of the image is parallel to the scan pattern of the transducer. C-scan presentations are produced with an automated data acquisition system, such as a computer controlled immersion scanning system. Typically, a data collection gate is established on the A-scan and the amplitude or the time-of-flight of the signal is recorded at regular intervals as the transducer is scanned over the test piece. Planar images can be generated on flat parts by tracking data to X-Y position, or on cylindrical parts by tracking axial and angular position. For conventional ultrasound, a mechanical scanner with encoders is used to track the transducer's coordinates to the desired index resolution The relative signal amplitude or the time-of-flight is displayed as a shade of gray or a color (b-mode representation) for each of the positions where data was recorded.

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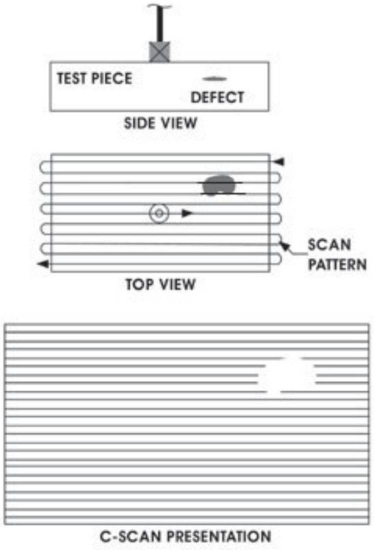
The C-scan presentation provides an image of the features that reflect and scatter the sound within and on the surfaces of the test piece.

Let us consider that the test piece has a discontinuity as shown in figure given below. The top view of the piece showing the direction of movement of the transducer i.e the scan pattern and the image constructed by plotting the brightness of the reflections at each of those scan points is also shown.



Again , it is to be noted that if the pulse strength is high and the depth of the discontinuity is small, then a 3D view of the discontinuity can also be obtained.

Medical Applications

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Therapeutic applications

Therapeutic applications use ultrasound to bring heat or agitation into the body. Therefore much higher energies are used than in diagnostic ultrasound. In many cases the range of frequencies used are also very different.

Ultrasound is sometimes used to clean teeth in dental hygiene.

Ultrasound sources may be used to generate regional heating and mechanical changes in biological tissue, e.g. in occupational therapy, physical therapy and cancer treatment. However the use of ultrasound in the treatment of musculoskeletal conditions has fallen out of favor

Focused ultrasound may be used to generate highly localized heating to treat cysts and tumors (benign or malignant), This is known as Focused Ultrasound Surgery (FUS) or High Intensity Focused Ultrasound (HIFU). These procedures generally use lower frequencies than medical diagnostic ultrasound (from 0.250 to 2 MHz), but significantly higher energies. HIFU treatment is often guided by MRI.

Focused ultrasound may be used to break up kidney stones by lithotripsy.

Ultrasound may be used for cataract treatment by phacoemulsification.

Additional physiological effects of low-intensity ultrasound have recently been discovered, e.g. its ability to stimulate bone-growth and its potential to disrupt the blood–brain barrier for drug delivery.

Procoagulant at 5–12 MHz.

SONOGRAPHY

Sonography is a diagnostic medical procedure that uses high frequency sound waves to produce dynamic visual images of organs, tissues, or blood flow inside the body. This type of procedure is called a sonogram.

There are several areas of specialization in the field of sonography:

Abdomen - evaluation of all the soft tissues, blood vessels and organs of the abdomen

Breast - frequently used to evaluate breast lumps and other abnormalities that are found with screening or mammography

Obstetrics/Gynecology - evaluation of the developing fetus and the female reproductive system

Cardiac - evaluation of the anatomy and function of the heart and related blood vessels

Vascular - evaluation of the blood flow through blood vessels

Neurosonology - evaluation of the brain and spinal cord in infants

Unlike X-rays, sonography does not use ionizing radiation.

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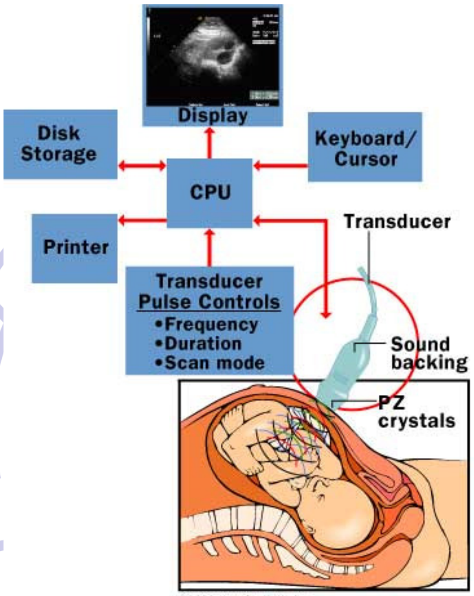
Sonography is increasingly being used in the detection and treatment of heart disease, heart attack, and vascular disease that can lead to stroke. It is also used to guide needles for tissue biopsies taken for testing under a microscope.



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A small device called a transducer is placed against the patient's skin near the body area to be imaged. The transducer works like a loudspeaker and microphone because it can transmit and receive sound.

The transducer sends a stream of high frequency sound waves into the body that bounce off the structures inside. The transducer detects sound waves as they bounce off the internal structures. These sounds are analyzed by a computer to make an image of the structure(s) on a computer screen or that can be recorded on videotape, CD, or DVD.

Detectors of nuclear radiation

Most of the nuclear reactions are accompanied by the emission of charged particles like α particles, protons, electron and radiation like

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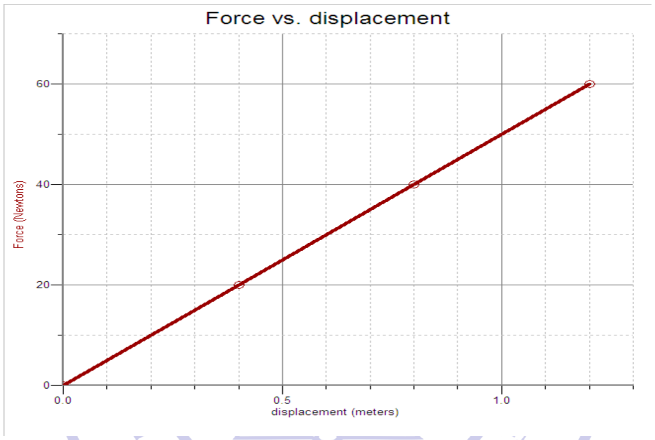
SOLVED PROBLEMS

1. A mass of 0.5 kg oscillates on the end of a spring on a horizontal surface with negligible friction according to the equation $x = A\cos(\omega t)$. The graph of *F vs. x* for this motion is shown below.



The last data point corresponds to the maximum displacement of the mass.

Determine the

(a) angular frequency *ω* of the oscillation,

(b) frequency *f* of oscillation,

(c) amplitude of oscillation,

(d) displacement from equilibrium position (*x* = 0) at a time of 2 s.

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Solution:

(a) We know that the spring constant *k* = 50 N/m from when we looked at this graph earlier. So,

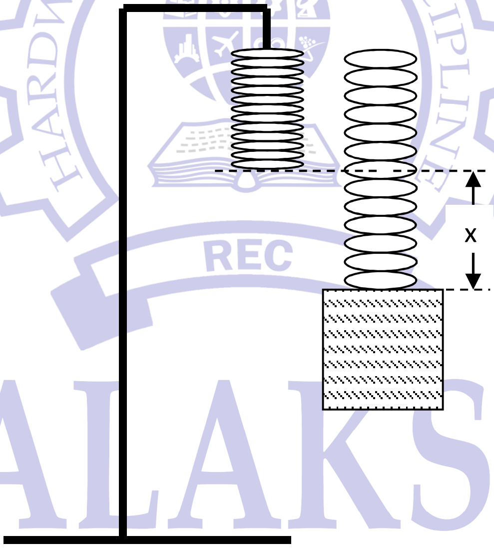
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\,N/m}{0.5\,kg}} = 10\frac{rad}{s}$$

(b) $f = \frac{\omega}{2\pi} = \frac{10\,rad/s}{2\pi} = 1.6\,Hz$

(c) The amplitude corresponds to the last displacement on the graph, A = 1.2 m.

(d) $x = A\cos(\omega t) = (1.2m)\cos[(10 rad/s)(2s)] = 0.5m$

2) A spring of constant k = 100 N/m hangs at its natural length from a fixed stand. A mass of 3 kg is hung on the end of the spring, and slowly let down until the spring and mass hang at their new equilibrium position.



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(a) Find the value of the quantity x in the figure above.

The spring is now pulled down an additional distance x and released from rest.

(b) What is the potential energy in the spring at this distance?

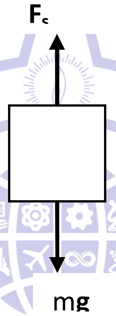
(c) What is the speed of the mass as it passes the equilibrium position?

(d) How high above the point of release will the mass rise?

(e) What is the period of oscillation for the mass?

Solution:

(a) As it hangs in equilibrium, the upward spring force must be equal and opposite to the downward weight of the block.



$$F_s = mg$$
$$kx = mg$$
$$x = \frac{mg}{k} = \frac{(3\,kg)(10\,m/s^2)}{100\ N/m} = 0.3\,m$$

(b) The potential energy in the spring is related to the displacement from equilibrium position by the equation

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(100\,N/m)(0.3m)^2 = 4.5\,J$$

(c) Since energy is conserved during the oscillation of the mass, the kinetic energy of the mass as it passes through the equilibrium position is equal to the potential energy at the amplitude. Thus,

$$K = U = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(4.5\,J)}{3\,kg}} = 1.7\,m/s$$

(d) Since the amplitude of the oscillation is 0.3 m, it will rise to 0.3 m above the equilibrium position.

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(e) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3\,kg}{100\,N/m}} = 1.1\,s$

3. At room temperature the ground state and the first excited state of Ruby are separated by 1.8 eV. Calculate the ratio of the number of atoms in the excited state to that in the ground state.

Given Eg = 1.8 eV = $1.8\ \times 1.6\ \times 10^{-19}$ J ; k = $1.38 \times 10^{-23}\ JK^{-1}$

Formula:

$$\frac{N_2}{N_1} = e^{-Eg/kT}$$

Calculation:

$$\frac{N_2}{N_1} = e^{-1.8\times 1.6\times 10^{-19}/(1.38\times 10^{-23}\times 300)}$$

(Ans. $e^{-70} \simeq 10^{-30}$)

4. Find the ratio of the probability of spontaneous emission to stimulated emission at 300 K for (a) microwave photons ($\boldsymbol{v = 10^{13} Hz.}$) and (b) optical photons ($\boldsymbol{v = 10^{15} Hz}$)

Given: $v_{microwave} = 10^{13} Hz$ and $v_{Optical\ photon} = 10^{15} Hz$

Formula: Ratio of spontaneous to stimulated emission = $e^{hv/kT} - 1\ \simeq e^{hv/kT}$

Calculation: $E_{microwave} = 6.63\ \times 10^{-34}\ \times 10^{13} = 6.63\ \times 10^{-21}\, J$

$E_{optical\ photon} = 6.63\ \times 10^{-34}\ \times 10^{15} = 6.63\ \times 10^{-19}\, J$

$$kT = 1.38 \times 10^{-23} \times\ 300 = 4.14 \times 10^{-21} J$$

Ratio of Spontaneous to stimulated

i) For microwave

$$e^{hv/kT} - 1 =\ e^{6.63\times 10^{-21}/4.14e-21} - 1 = 3.96$$

ii) For optical photon

$$e^{hv/kT} - 1 =\ e^{6.63\times 10^{-19}/4.14e-21} - 1 \backsim 10^{65}$$

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5. Calculate the numerical aperture, acceptance angle and the critical angle of a fiber having refractive index = 1.5 and a cladding refractive index = 1.45.

Solution:

$$\text{Numerical aperture (NA)} = \sqrt{{n_1}^2 - {n_2}^2}$$

$$= \sqrt{1.50^2 - 1.50^2}$$

$$= 0.3840$$

$$\text{Acceptance angle } \theta_a(\max) = sin^{-1}\,(NA)$$

$$= sin^{-1}\,(0.384)$$

$$= 22.59^0$$

$$\text{Acceptance angle } \theta_a(\max) = sin^{-1}\,(NA)$$

$$= sin^{-1}\,\frac{n_2}{n_1}$$

$$= sin^{-1}\,\frac{1.45}{1.50}$$

$$= 75^0 16'$$

6. Calculate the numerical aperture, acceptance angle of a fiber with a core index of 1.54 and cladding index of 1.50 when the fiber is inside water of refractive index 1.33.

Solution:

$$\text{Numerical aperture (NA)inside water} \sin\theta_i = \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0}$$

$$= \frac{\sqrt{1.54^2 - 1.50^2}}{1.33}$$

$$= 0.262$$

$$\text{Acceptance angle inside water } \theta_i(\max) = sin^{-1}\,(NA)$$

$$= sin^{-1}\,(0.262)$$

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$$= 15.18^0$$

7. A certain optical fibre has an attenuation of 3.5 dB/km at 850 mm. If 0.5 mW of optical power is initially launched into the fibre, what is the power level in mW after 4 km.

Attenuation α = 3.5 dB/km.
Initial power level, P_i = 0.5 mW.
Length of the cable, L = 4 km.

Solution:

$$\text{Attenuation loss } \alpha = \left(\frac{10}{L}\right) \log_{10} \left(\frac{P_{in}}{P_{out}}\right) \text{ dB}$$

$$(10) \log_{10} \left(\frac{0.5\ mW}{P_{out}}\right) \text{ dB} = (3.5 \text{ dB/km}) (4 \text{ km})$$

$$= 14 \text{ dB}$$

$$P_{Out} = \left(\frac{0.5mW}{25.11}\right)$$

$$= 19.9\ \mu W.$$

8. An optical signal has lost 85% its power after traversing 500 m of fibre. What is the loss in dB/km of this fibre?

Solution: Let the initial power be P_I = 1 mW

$$P_O = 85\%\ P_i = 0.85 \text{ mW}$$

$$L = 500 \text{ m} = 0.5 \text{ km}.$$

$$\text{Attenuation loss } \alpha = \left(\frac{10}{L}\right) \log_{10} \left(\frac{P_{in}}{P_{out}}\right) \text{ dB} = \left(\frac{10}{0.5}\right) \log_{10} \left(\frac{1}{0.85}\right) \text{ dB}$$

$$= 1.41 \text{ dB/km}$$

9. Calculate the refractive indices of core and cladding material of an optical fiber if its numerical aperture is 0.22 and relative refractive indices is 0.012.

Solution:

$$NA = n_1\sqrt{2\Delta}$$

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Given $NA = 0.22$

$\boldsymbol{\Delta} = 0.012$

$$n_1 = \frac{NA}{\sqrt{2\Delta}}$$

$$= \frac{0.22}{\sqrt{2X0.012}}$$

$$= 1.42$$

$n_1 - n_2 = \boldsymbol{\Delta}\, n_1$

$n_2 = n_1 - \boldsymbol{\Delta}\, n_1$

$n_1 (1 - \boldsymbol{\Delta}) = 1.42\ (1 - 0.012)$

$= 1.403$

Hence $n_2 = 1.42$ and $n_2 = 1.403$

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UNIT 5

NUCLEAR AND PARTICLE PHYSICS

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UNIT 5

NUCLEAR AND PARTICLE PHYSICS

5.1 RADIOACTIVITY

The observation by Herri beequerel (1896) of the blackening of the photographic plate placed unintentionally close to a uranium preparation led perhaps to greatest discovery of the century the name radioactivity was proposed by a polish pupil of Becquerel, marie Sklodowska, who later became the illustrious scientist Marie Curie, the recipient of two Nobel prizes.

RADIOACTIVITY ELEMENTS

An element is said to be radioactive if the nuclei of its atoms keep disintegrating spontaneously transforming thereby into a different element in the process nuclear particles radiation.

5.2 TYPES OF RADIOACTIVE DECAY

Type	**Nuclear reaction**	**Example**
α-decay	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$	${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$
β^-decay	${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}Y + \beta + \gamma$	${}^{32}_{15}\beta \rightarrow {}^{32}_{16}S + \beta + \gamma$
	$n \rightarrow P + \beta + \gamma$	
β+ decay	${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}Y + \beta^{+} + \gamma$	${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + \beta + \gamma$
	$\mathrm{P} \rightarrow n + \beta^{+} + \gamma$	
Electron Capture	${}^{A}_{Z}X + e \rightarrow {}_{Z-1}{}^{A}Y + \gamma$	${}^{55}_{26}Fe + e \rightarrow {}^{55}_{25}Mn + \gamma$
Isometric Transition	${}^{Am}_{Z}X \rightarrow {}^{A}_{Z}X + \gamma$	${}^{60m}_{27}CO \rightarrow {}^{60}_{27}CO + \gamma$

m – metastable state

5.3 GENERAL CHARACTERISTICS OF RADIOACTIVE DECAY

(i) MASS LOSS AND ENERGY RELEASE

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The sum of masses of the products of nuclear decay is always than the mass of the present atom, the equivalent of the mass loss appears as the recoil energy of daughter atom and kinetic energy of the particles and the energy of the photon emitted.

$$P^{32} \rightarrow S + \beta^- + \gamma^-$$

Mass /u 31.973 > 31.972 $\Delta m = 0.001834u^*$

$\Delta m - energy\ released$

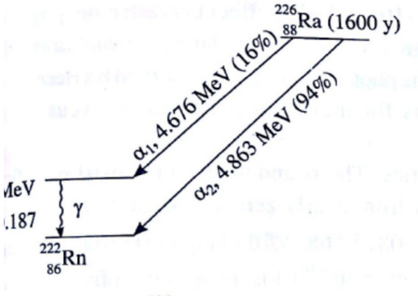
(ii) NUCLEAR RADIATIONS

During the decay of radioactive nucleus, either helium ions (α decay) or electrons βdecay or positrons βdecay are emitted. These particles are in many cases accompanied by pure electromagnetic radiation in the form of γ photons. These radiations are of high energy ($10^3 - 10^6$ eV)and have high penetrating power through matter.

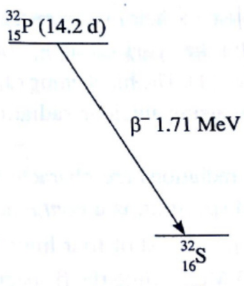
$R_\gamma >> R_\beta >> R_\alpha$

The ionizing power is in the reverse order

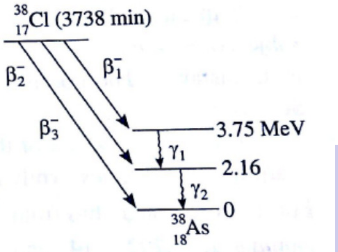
$I\alpha > I\beta > I\gamma$



(a) α decay of ^{236}Ra



(b) β⁻ decay of ^{32}P

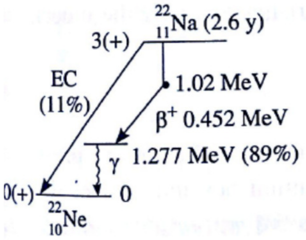


(c) β⁻ decay of ^{38}Cl,

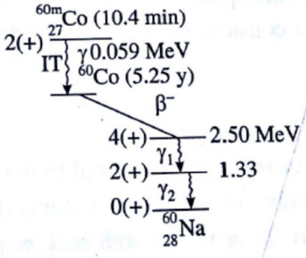
$\beta_1 = 1.11$ MeV (31%)

$\beta_2 = 2.77$ „ (16%)

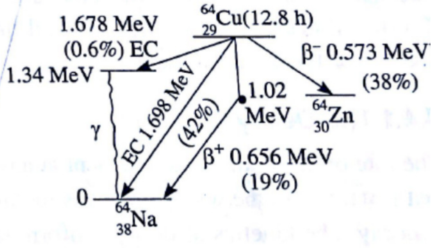
$\beta_3 = 4.81$ „ (53%)



(d) β⁺ decay of ^{22}Na spin and parity shown on right extreme



(e) I.T. and β⁻ decay of ^{60}Co



(f) Branched decay of ^{64}Cu

5.4 PROBING BY ISOTOPES

If we know the precise mechanism of a chemical, physical or biochemical reaction, how the plants draw their nutrients from the soil, However as it is possible to know what goes on from moment to moment in the sub micro world of atoms and molecules, we have to depend on our best guesses based on indirect gross observations and empire deductions.

Isoptopes of the same elements are practically indistinguishable from another. The only difference which 120 radioisotopes make is in the emission of signals which can be measured with great precision. Thus one can monitor the movements of the reactant atom alone different intermediate stages to the end product. It is possible by the use of radioisotopes as tracers.

Innumerable application of radioisotopes in research, industry, medicine and agricultural have been developed. We shall present here only some selected application in each area.

5.5 PREPARATION OF RADIOISOTOPES

We shall describe how some typical reactions involved in the formation of a few selected radioisotopes and some simple methods of their preparation.

a. **Tritium or hydrogen -3 (Z :12.33 y:decay:^{3}H +^{3}He+β+γ)**

Significant amount of 3H are formed continuously in nature by the interaction of fast neutrons of cosmic origin (flux 2.6 ncm^{-2}s^{-1}) with the nitrogen of the atmosphere.

$$^{14}N + n \rightarrow {}^{3}H + {}^{12}C$$

In due course the T (3H) forms HT and gets oxidized into initiated water HTO and through rains reaches the seas, rivers, lakes and such other large surfaces of water open to the atmosphere.

For large scale needs of 3H for research and nuclear fusion reactions it is obtained by ne reaction 6Li(n, α) ^{3}H.

The overall yield of the 3H tagged molecule is reasonably high (10-50%) only the tugging is a random process.

b. **Carbon – 14 (Z:5730; y:decay: ^{14}C → ^{14}N + P**

Large amount of ^{14}C are formed continuous in nature by an alternative interaction of fast neutrons of cosmic origin with the atmosphere nitrogen

$$^{14}N + n \rightarrow {}^{14}C + P$$

Within minutes and hours the ^{14}C changes into $^{14}CO_2$ which together with ordinary CO_2 of atmosphere is taken up by living plants to provide the food for their growth by a photosynthetic process. The ^{14}C/^{12}C ratio in any living plants to provide the food for their growth by a photosynthetic process. The ^{14}C/^{12}C ratio in any living plant for animal is the equilibrium mass ratio of 1.6×10^{-12} yielding 15.8 ± 0.3 disintegrations min^{-1}g^{-1} (total carbon). For large scale needs of ^{14}Cused in the preparation of different organic substance

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labeled with this isotope, a substance rich in nitrogen, as beryllium or aluminum nitride. The desire organic substance labeled with ^{14}C can be synthesized by releasing $^{14}CO_2$ from the barium carbonate.

c. **Sodium – 22 (Z:2.6 y:decay : $^{22}Na \rightarrow {}^{22}Ne + \beta^+ + \gamma$**

^{22}Na is a cyclotron or similar accelerator product obtained by bombarding magnesium with 8.5 MeV deuterons by the reaction 24 Mg (d, α) 22 Na. The irradiated magnesium is dissolved in dilute hydrochloric acid and the solution electrolyzed at 12V between a platinum anode and resolving mercury cathode as shown in Fig…

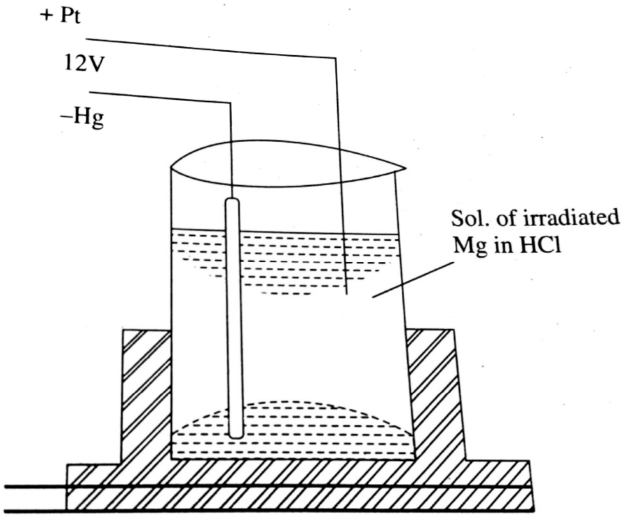

Fig. 5.1 Separation of ^{22}Na by the electrolysis of a solution in hydrochloric acid of magnesium subjected to (d, α)

The ^{22}Na goes to the mercury and the amalgam formed can be subsequently decomposed to yield $^{22}NaOH$ solution.

Sodium – 24 (Z: 15.02 h y:decay : $^{24}Na \rightarrow {}^{24}Mg + \beta^+ + \gamma^-$

Obtainable by (n, γ) reaction on any sodium compound.

d. **Phosphorous – 32 (Z: 14.28 d y:decay : $^{32}P \rightarrow {}^{32}S + \beta^-$)**

Two modes are in vogue for obtaining radiophosphorus. (i) by (n, γ) reaction on ordinaryred phosphorus 32P isotopic abundance: 100%) This is less expensive but the product would be of a low specific activity. The radio phosphorus is converted into any of the several reactive specific $P2O_5$, PCl_3 or $PsCl_3$) (ii) By (n, p) reaction on 32S (isotopic abundant product through carrier free is mixed with 33P formed from 33S (Isotopic abundance : 0.75% where high specific activity is needed this procedure is adopted. The unreacted sulphur sublimed off at 600°C and the residue extracted in an acidic medium.

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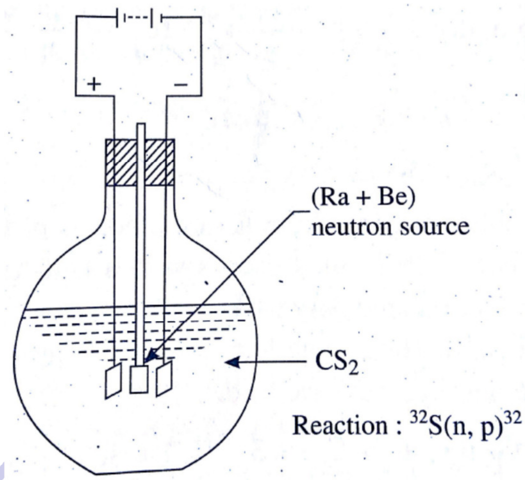

Fig. 5.2 Formation of ^{32}P and its collection on charged plates

e. **Sulphur – 35 (Z :87.5d : decay : $^{35}S \rightarrow {}^{35}Cl + \beta^- + \gamma$)**

^{35}S is formed when sodium chloride is irradiated by high flux neutrons, as in a reactor. When the reaction ^{35}Cl (n,P) ^{35}S occurs. Following other reactions also occur at the same time resulting in the formation of corresponding radioisotopes:
^{23}Na (N,γ) ^{24}Na; ^{37}Cl (n,γ)^{38}Cl (n,α) ^{32}P add to a negligible extent ^{35}Cl (n,γ) ^{36}Cl.
The ^{35}S is readily extracted in the form of carrier free $H_2{}^{35}SO_4$ as follows: The irradiated sodium chloride is allowed to stand for about a week when the 24 Na (Z:15h) and 38 Cl (Z:37 min) decay completely (the yields of the long lived 36 Cl and 32p being negligible) it is then heated in a platinum boat in a slow current of pure dry hydrogen and the exit gas is made to buddle through some bromine water.
When the following reaction occur

$$^{35}S \rightarrow H_2{}^{35}S \rightarrow H_2{}^{35}SO_4$$

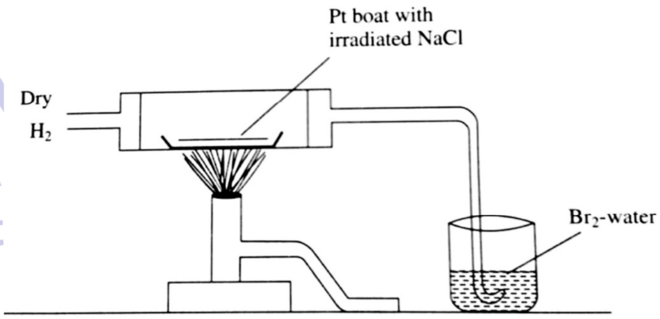

Fig. 5.3 Separation of ^{35}S is fixed as Thiourea making it react with cynamide

Alternatively, the $H_2{}^{35}S$ is fixed as Thiourea by making it react with cynamide.

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SZILARD CHALMERS EFFECT ;

An effect discovered by L. Szilard (1898–1964) and T. A. Chalmers in 1934; it has been used to separate radioactive products in a nuclear reaction involving the absorption of a neutron and the emission of gamma rays. If a material absorbs a neutron and subsequently emits a gamma ray, the emission of the gamma ray causes the nucleus to recoil. Frequently, the recoil energy is sufficient to break the chemical bond between the atom and the molecule of which it forms part. Thus, although the atom that has absorbed the neutron is an isotope of the original atom it is in a different form chemically, enabling separation to take place.

For example, if an aqueous solution of sodium chlorate ($NaClO_3$) is subjected to bombardment by slow neutrons, the Cl^{37} is converted to Cl^{38}, with many of the Cl^{38} atoms breaking from the chlorate and moving into the solution in the form of chloride ions. This is an example of a 'hot atom' reaction. The Cl^{38} can be precipitated out using silver nitrate.

RADIOCHEMICAL PRINCIPLES IN THE USE OF TRACERS

Isotopes of a chemical element differ only in the mass number. For example, the isotopes of hydrogen can be written as ^{1}H, ^{2}H and ^{3}H, with the mass number superscripted to the left. When the atomic nucleus of an isotope is unstable, compounds containing this isotope are radioactive. Tritium is an example of a radioactive isotope.

The principle behind the use of radioactive tracers is that an atom in a chemical compound is replaced by another atom, of the same chemical element. The substituting atom, however, is a radioactive isotope. This process is often called radioactive labeling. The power of the technique is due to the fact that radioactive decay is much more energetic than chemical reactions. Therefore, the radioactive isotope can be present in low concentration and its presence detected by sensitive radiation detectors such as Geiger counters and scintillation counters.

There are two main ways in which radioactive tracers are used

1. When a labeled chemical compound undergoes chemical reactions one or more of the products will contain the radioactive label. Analysis of what happens to the radioactive isotope provides detailed information on the mechanism of the chemical reaction.
2. A radioactive compound is introduced into a living organism and the radio-isotope provides a means to construct an image showing the way in which that compound and its reaction products are distributed around the organism.

Production

The commonly used radioisotopes have short half lives and so do not occur in nature. They are produced by nuclear reactions. One of the most important processes is absorption of a

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neutron by an atomic nucleus, in which the mass number of the element concerned increases by 1 for each neutron absorbed. For example,

$$^{13}C + n \rightarrow {}^{14}C$$

In this case the atomic mass increases, but the element is unchanged. In other cases the product nucleus is unstable and decays, typically emitting protons, electrons (beta particle) or alpha particles. When a nucleus loses a proton the atomic number decreases by 1. For example,

$$^{32}S + n \rightarrow {}^{32}P + p$$

Neutron irradiation is performed in a nuclear reactor. The other main method used to synthesize radioisotopes is proton bombardment. The proton are accelerated to high energy either in a cyclotron or a linear accelerator

Isotopes

Hydrogen:

Tritium is produced by neutron irradiation of ^{6}Li

$^{6}Li + n \rightarrow {}^{4}He + {}^{3}H$

Tritium has a half-life 4,500±8 days (approximately 12.32 years), and it decays by beta decay. The electrons produced have an average energy of 5.7 keV. Because the emitted electrons have relatively low energy, the detection efficiency by scintillation counting is rather low. However, hydrogen atoms are present in all organic compounds, so tritium is frequently used as a tracer in biochemical studies.

Carbon:

^{11}C decays by positron emission with a half-life of ca. 20 min. ^{11}C is one of the isotopes often used in positron emission tomography.

^{14}C decays by beta-decay, with a half-life of 5730 y. It is continuously produced in the upper atmosphere of the earth so it occurs at a trace level in the environment. However, it is not practical to use naturally-occurring ^{14}C for tracer studies. Instead it is made by neutron irradiation of the isotope ^{13}C which occurs naturally in carbon at about the 1.1% level. ^{14}C has been used extensively to trace the progress of organic molecules through metabolic pathways.

Nitrogen:

^{13}N decays by positron emission with a half-life of 9.97 min. It is produced by the nuclear reaction

$^{1}H + {}^{16}O \rightarrow {}^{13}N + {}^{4}He$

^{13}N is used in positron emission tomography (PET scan).

Oxygen:

^{15}O decays by positron emission with a half-life of 122 sec. It is used in positron emission tomography

Fluorine:

^{18}F decays by emission with a half-life of 109 min. It is made by proton bombardment of ^{18}O in a cyclotron or linear particle accelerator. It is an important isotope in the radiopharmaceutical industry. It is used to make labeled fluorodeoxyglucose (FDG) for application in PET scans

Phosphorus:

^{32}P is made by neutron bombardment of ^{32}S

$^{32}S + n \rightarrow ^{32}P + p$

It decays by beta decay with a half-life of 14.29 days. It is commonly used to study protein phosphorylation by kinases in biochemistry.

^{33}P is made in relatively low yield by neutron bombardment of ^{31}P. It is also a beta-emitter, with a half-life of 25.4 days. Though more expensive than ^{32}P, the emitted electrons are less energetic, permitting better resolution in, for example, DNA sequencing.

Both isotopes are useful for labeling nucleotides and other species that contain a phosphate group.

Sulfur:

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^{35}S is made by neutron bombardment of ^{35}Cl

$^{35}Cl + n \rightarrow ^{35}S + p$

It decays by beta-decay with a half-life of 87.51 days. It is used to label the sulfur-containing amino-acids methionine and cysteine. When a sulfur atom replaces an oxygen atom in a phosphate group on a nucleotide a thiophosphate is produced, so ^{35}S can also be used to trace a phosphate group.

NUCLEAR MEDICINE

a) Nuclear Medicine is a branch of medicine that uses radiation to provide information about a person’s anatomy and the functioning of specific organs. In most cases, the information enables physicians to provide a quick, accurate diagnosis of conditions such as cancer, heart disease, thyroid disorders and bone fractures. In some cases, radiation is used to treat the condition.

b) More specifically, nuclear medicine is a part of molecular imaging because it produces images that reflect biological processes that take place at the cellular and subcellular level.

Procedure

a) Introducing small amount of radiopharmaceuticals into the body by
   - Injection
   - Swallowing
   - Inhalation

b) The amount of radiopharmaceuticals used is carefully selected to provide the least amount of radiation exposure to the patient but ensure an accurate test.

c) Time taken for the radionuclide to travel to the target.

d) Taking pictures of the body by special cameras (PET-,SPECT or gamma camera) .

e) Detecting the emitted gamma rays from inside the body by the gamma camera , converting it into an electrical signal and send to a computer.

f) Analysis with the computer.

Radiotherapy

a) Rapidly dividing cells are particularly sensitive to damage by radiation.

b) For this reason, some cancerous growths can be controlled or eliminated by irradiating the area containing the growth.

c) Many therapeutic procedures are palliative, usually to relieve pain. For instance, strontium-89 and (increasingly) samarium 153 are used for the relief of cancer-induced bone pain. Rhenium-186 is a newer product for this.

d) Treating leukemia may involve a bone marrow transplant, in which case the defective bone marrow will first be killed off with a massive (and otherwise lethal) dose of radiation before being replaced with healthy bone marrow from a donor.

TAT

a) A new field is Targeted Alpha Therapy (TAT), especially for the control of dispersed cancers. The short range of very energetic alpha emissions in tissue means that a large fraction of that irradiative energy goes into the targeted cancer cells, once a carrier has taken the alpha-emitting radionuclide to exactly the right place.

b) An experimental development of this is Boron Neutron Capture Therapy using boron-10 which concentrates in malignant brain tumors. The patient is then irradiated with thermal neutrons which are strongly absorbed by the boron, producing high-energy alpha particles which kill the cancer.

ADVANTAGES

a) It provides doctors with information about both structure and function.
b) It's a way to gather the medical information that would otherwise be unavailable, require surgery, or necessitate more expensive diagnostic tests.
c) Often identify abnormalities very early in the progress of a disease, long before many medical problems are apparent with other diagnostic test.
d) It determines the presence of a disease based on biological changes rather than changes in anatomy.
e) Combination with CT scan can give the image of both bone and soft tissue.
f) It's extremely safe because
    - The radioactive tracers, or radiopharmaceuticals, used are quickly eliminated from the body through its natural functions.
    - The tracers used rapidly lose their radioactivity.
    - The dose of radiation necessary for a scan is very small.

INTERACTION OF CHARGED PARTICLES WITH MATTER

It is well known that all matter is comprised of atoms. But subatomically, matter is made up of mostly empty space. For example, consider the hydrogen atom with its one proton, one neutron, and one electron. The diameter of a single proton has been measured to be about 10^{-15} meters. The diameter of a single hydrogen atom has been determined to be 10^{-10}meters, therefore the ratio of the size of a hydrogen atom to the size of the proton is 100,000:1. Consider this in terms of something more easily pictured in your mind. If the nucleus of the atom could be enlarged to the size of a softball (about 10 cm), its electron would be approximately 10 kilometers away. Therefore, when electromagnetic waves pass through a material, they are primarily moving through free space, but may have a chance encounter with the nucleus or an electron of an atom.

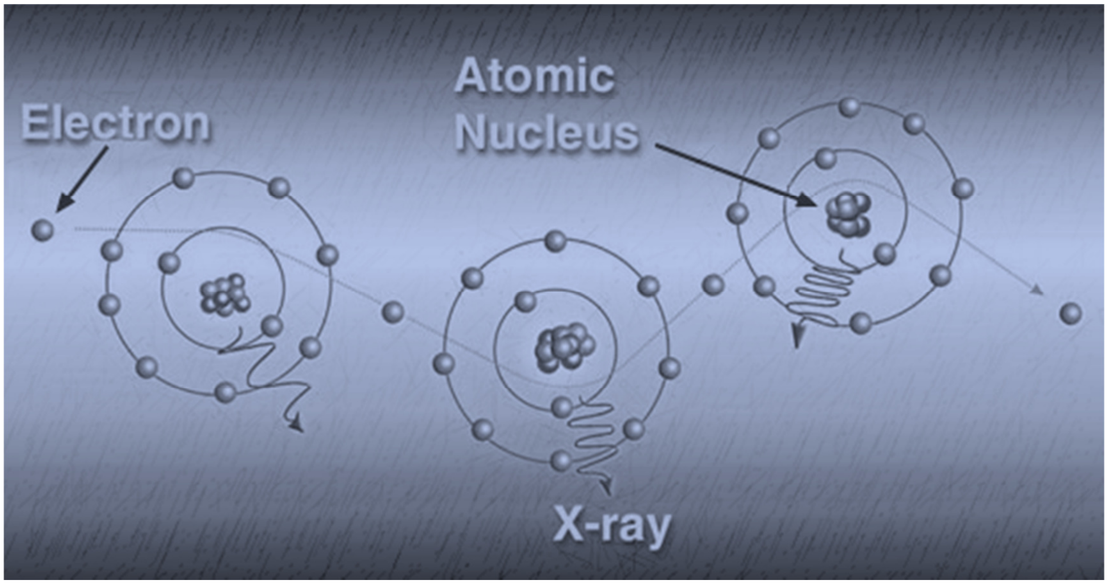
Because the encounters of photons with atom particles are by chance, a given photon has a finite probability of passing completely through the medium it is traversing. The probability that a photon will pass completely through a medium depends on numerous factors including the photon's energy and the medium's composition and thickness. The more densely packed a medium's atoms, the more likely the photon will encounter an atomic particle. In other words, the more subatomic particles in a material (higher Z number), the greater the likelihood that interactions will occur Similarly, the more material a photon must cross through, the more likely the chance of an encounter.

When a photon does encounter an atomic particle, it transfers energy to the particle. The energy may be reemitted back the way it came (reflected), scattered in a different direction or transmitted forward into the material. Let us first consider the interaction of visible light. Reflection and transmission of light waves occur because the light waves transfer energy to the electrons of the material and cause them to vibrate. If the material is transparent, then the vibrations of the electrons are passed on to neighboring atoms through the bulk of the material and reemitted on the opposite side of the object. If the material is opaque, then the vibrations of the electrons are not passed from atom to atom through the bulk of the material, but rather the electrons vibrate for short periods of time and then reemit the energy as a reflected light wave.

The light may be reemitted from the surface of the material at a different wavelength, thus changing its color.

X-Rays and Gamma Rays

X-rays and gamma rays also transfer their energy to matter though chance encounters with electrons and atomic nuclei. However, X-rays and gamma rays have enough energy to do more than just make the electrons vibrate. When these high energy rays encounter an atom, the result is an ejection of energetic electrons from the atom or the excitation of electrons. The term "excitation" is used to describe an interaction where electrons acquire energy from a passing charged particle but are not removed completely from their atom. Excited electrons may subsequently emit energy in the form of x-rays during the process of returning to a lower energy state.



Each of the excited or liberated electrons goes on to transfer its energy to matter through thousands of events involving interactions between charged particles. With each interaction, the energy may be directed in a different direction. The higher the energy of a photon, the more likely the energy will continue traveling in the same direction. As the radiation moves from point to point in matter, it loses its energy through various interactions with the atoms it encounters. If the radiation has enough energy, it may eventually make it through the material.

Specific ionization

As penetrating radiation moves from point to point in matter, it loses its energy through various interactions with the atoms it encounters. The rate at which this energy loss occurs depends upon the type and energy of the radiation and the density and atomic composition of the matter through which it is passing.

The various types of penetrating radiation impart their energy to matter primarily through excitation and ionization of orbital electrons. The term "excitation" is used to describe an interaction where electrons acquire energy from a passing charged particle but are not removed completely from their atom. Excited electrons may subsequently emit energy in the form of x-rays during the process of returning to a lower energy state. The term "ionization" refers to the complete removal of an electron from an atom following the transfer of energy from a passing charged particle. In describing the intensity of ionization, the term "specific ionization" is often used. This is defined as the number of ion pairs formed per unit path length for a given type of radiation.

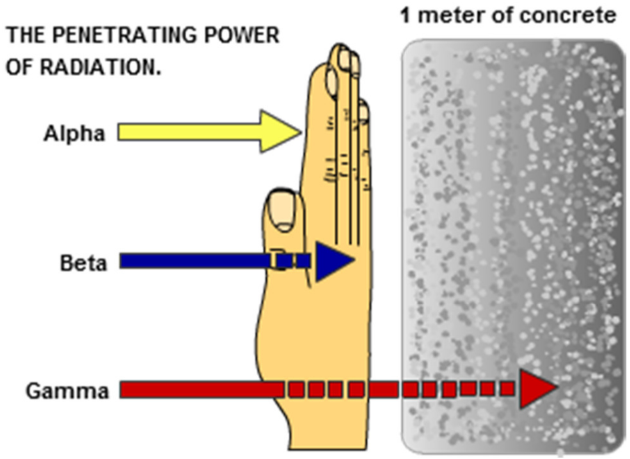


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Fig.1

Because of their double charge and relatively slow velocity, alpha particles have a high specific ionization and a relatively short range in matter (a few centimeters in air and only fractions of a millimeter in tissue). Beta particles have a much lower specific ionization than alpha particles and, generally, a greater range.

Gamma-rays, x-rays, and neutrons are referred to as indirectly ionizing radiation since, having no charge, they do not directly apply impulses to orbital electrons as do alpha and beta particles. Electromagnetic radiation proceeds through matter until there is a chance of interaction with a particle. If the particle is an electron, it may receive enough energy to be ionized, whereupon it causes further ionization by direct interactions with other electrons. As a result, indirectly ionizing radiation (e.g. gamma, x-rays, and neutrons) can cause the liberation of directly ionizing particles (electrons) deep inside a medium. Because these neutral radiations undergo only chance encounters with matter, they do not have finite ranges, but rather are attenuated in an exponential manner. In other words, a given gamma ray has a definite probability of passing through any medium of any depth.

Neutrons lose energy in matter by collisions which transfer kinetic energy. This process is called moderation and is most effective if the matter the neutrons collide with has about the same mass as the neutron. Once slowed down to the same average energy as the matter being interacted with (thermal energies), the neutrons have a much greater chance of interacting with a nucleus. Such interactions can result in material becoming radioactive or can cause radiation to be given off.

Linear energy transfer

Linear energy transfer (LET) is the amount of energy that an ionizing particle transfers to the material traversed per unit distance. It describes the action of radiation into matter. It is identical to the retarding force acting on a charged ionizing particle travelling through the matter. By definition, LET is a positive quantity. LET depends on the nature of the radiation as well as on the material traversed.

A high LET will attenuate the radiation more quickly, generally making shielding more effective and preventing deep penetration. On the other hand, the higher concentration of deposited energy can cause more severe damage to any microscopic structures near the particle track. If a microscopic defect can cause larger-scale failure, as is the case in biological cells and microelectronics, the LET helps explain why radiation damage is sometimes disproportionate to the absorbed dose. Dosimetry attempts to factor in this effect with radiation weighting factors.

Linear energy transfer is closely related to stopping power, since both equal the retarding force. The unrestricted linear energy transfer is identical to linear electronic stopping power, as discussed below. But the stopping power and LET concepts are different in the respect that total stopping power has the nuclear stopping power component, and this component does not cause electronic excitations. Hence nuclear stopping power is not contained in LET.

The appropriate SI unit for LET is the newton, but it is most typically expressed in units of kiloelectronvolts per micrometre (keV/μm) or megaelectronvolts per centimetre (MeV/cm).

Range

In passing through matter, charged particles ionize and thus lose energy in many steps, until their energy is (almost) zero. The distance to this point is called the **range** of the particle. The range depends on the type of particle, on its initial energy and on the material through which it passes.

For example, if the ionising particle passing through the material is a positive ion like an alpha particle or proton, it will collide with atomic electrons in the material via Coulombic interaction. Since the mass of the proton or alpha particle is much greater than that of the electron, there will be no significant deviation from the radiation's incident path and very little kinetic energy will be lost in each collision. As such, it will take many successive collisions for such heavy ionising radiation to come to a halt within the stopping medium or material. Maximum energy loss will take place in a head-on collision with an electron.

Since large angle scattering is rare for positive ions, a range may be well defined for that radiation, depending on its energy and charge, as well as the ionisation energy of the stopping medium. Since the nature of such interactions is statistical, the number of collisions required to bring a radiation particle to rest within the medium will vary slightly with each particle (i.e., some may travel further and undergo fewer collisions than others). Hence, there will be a small variation in the range, known as **straggling**.

The energy loss per unit distance (and hence, the density of ionization), or stopping power also depends on the type and energy of the particle and on the material. Usually, the energy loss per unit distance increases while the particle slows down. The curve describing this fact is called the Bragg curve. Shortly before the end, the energy loss passes through a maximum, the Bragg Peak, and then drops to zero (see the figures in Bragg Peak and in stopping power). This fact is of great practical importance for radiation therapy.

The range of alpha particles in ambient air amounts to only several centimeters; this type of radiation can therefore be stopped by a sheet of paper. Although beta particles scatter much more than alpha particles, a range can still be defined; it frequently amounts to several hundred centimeters of air.

Bremsstrahlung

Bremsstrahlung is a German word meaning braking radiation. It refers to the process in which decelerating charged particles emit electromagnetic radiation. All charged particles can emit this kind of radiation provided they have enough energy. Generally speaking if a charged particle has energy much greater than its rest energy ($E_{rest} = E_0 = m_0c^2$, where m_0 is its rest mass) emits Bremsstrahlung if it encounters resistance while moving through a medium.

In pure Bremsstrahlung process there are no direct electronic or nuclear transitions involved. However the radiation emitting particle may excite or ionize atoms and excite nuclei of the medium as it decelerates. These excitations and ionizations may lead to emission of other particles, such as x-rays and γ-rays with characteristic energy peaks in the spectrum. These peaks are generally superimposed on the continuous Bremsstrahlung spectrum and are therefore clearly distinguishable. The most common example of this phenomenon is the emission of x-rays. The electrons, as they strike the anode, emit not only characteristic x-rays but also Bremsstrahlung. In fact, as the electron has a very small mass as compared to other charged particles, its Bremsstrahlung is one of the most commonly encountered radiations in laboratories.

If an electron is accelerated through a potential V (as in x-ray machines), the maximum energy it can attain is given by

$$E_{max} = eV, \tag{1}$$

where e is the unit electronic charge. The maximum energy of the Bremsstrahlung in the form of photons that this electron can emit will then also be equal to eV , that is

$$E_{brems} \leq E_{max} = eV. \quad (2)$$

Since Bremsstrahlung is emitted in the form of photons having energy E = hν = hc/λ therefore we can write the above equation as

$$h\,c/\lambda \leq eV$$

$$\Rightarrow \lambda \geq hc\; eV\;. \quad (3)$$

Hence we can associate a minimum wavelength λ_{min} with the process below which there will be no Bremsstrahlung photons emitted.

$$\lambda_{min} = hc/eV \quad (4)$$

λ_{min} is also called the cutoff wavelength for Bremsstrahlung

Example: Compute the cutoff Bremsstrahlung wavelength for an electron moving under the influence of a potential of 40 kV.

Solution: From equation (4) to compute the cutoff wavelength.

$$\lambda_{min} = hc\; eV$$

$$= (6.626 \times 10^{-34})\,(2.99 \times 10^{8})\,/(1.602 \times 10^{-19})(40 \times 10^{3})\text{ m}$$

$$= 30.91\text{ fm}$$

Annihilation

When a particle interacts with its antiparticle, the result is the annihilation of both and generation of other particles. The most common and oft quoted example of this process is the electron-positron annihilation. Positron is the antiparticle of electron having all the characteristics of an electron except for the unit electrical charge, which in its case is positive. When it comes very close to an electron, the two annihilate each other producing other particles. Depending on their energies, different particles can be generated during this process. At low to moderate energies, only photons are produced while at high energies other particles such as Z bosons can be produced.

The annihilation process in itself does not have any threshold energy. That is, the process can also happen even if the electron and positron are at rest. However to produce particles other than photons, the electron and the positron must be allowed to collide with each other at very high energies. This is actually done, for example at Large Electron Positron collider (or LEP) at CERN in Switzerland where particles are accelerated to center of mass energies reaching at 45 – 100 GeV before collision. The result of the collision is generation of large number of particles, which are then tracked down and identified to get clues about the fundamental particles and their interactions. At low electron-positron energies, at least two photons are produced in the annihilation process. The reason why only one photon can not be emitted lies in the law of conservation of momentum. According to this law, the total momentum of the emitted photons must be equal to that of the total momentum of electron and positron. Now, if the electron and the positron move in opposite directions with equal kinetic energies or are at rest before annihilation, the net momentum before collision will be zero. This implies that after the annihilation the total momentum must also be zero. We can not have a zero net momentum with only one photon and hence we conclude that at least two photons must be produced. Hence by using a simple argument, we have set a lower limit on the number of photons produced.

Let us consider that an electron-positron annihilation process produces only two equal energy photons. If the net momentum before collision was zero then the photons must travel in opposite direction to each other since only then the momentum conservation can be guaranteed. The energy these photons carry can be deduced from the law of conservation of energy, which states that the total energy of the system before and after collision must be equal. The reader may recall that the total energy of a particle is the sum of its kinetic and rest mass energies, such as

$$E = T + m_0c^2, \qquad (1)$$

where T is the kinetic energy, m_0 is the rest mass, and c is the velocity of light. The total energy before collision can then be written as

$$E_{total} = Ee + Ee+$$

$$= Te + m_0c^2 + Te^+ + m_0c^2$$

$$= Te + Te^+ + 2m_0c^2,$$

Whereboth the electron and the positron have equal rest masses. The kinetic energies of the two particles will be zero if they were at rest before the annihilation. Hence the total energy before the collision will be

$$Etotal = 2m_0c^2 = 2 \times 511 \text{ keV}.$$

This shows that to conserve energy, each of the two photons produced must carry an energy of 511 keV. Hence we can conclude that if the conservation of momentum and energy are to be satisfied then at the minimum there must be two photons each carrying 511 keV of energy and travelling in opposite directions.

Gamma ray spectrometry

Gamma ray spectrometry is an analytical method that allows the identification and quantification of gamma emitting isotopes in a variety of matrices. In one single measurement and with little sample preparation, gamma ray spectrometry allows to detect several gamma emitting radionuclei in the sample. The measurement gives a spectrum of lines , the amplitude of which is proportional to the activity of the radionuclide and its position on the horizontal axis gives an idea on its energy. Applications of gamma ray spectrometry include:

- monitoring in nuclear facilities,
- health physics,

- nuclear medicine,
- research in materials,
- bioscience,
- environmental science, and
- industrial uses of radioisotopes.

Because of their deep penetrating power, γ-rays are routinely used for nuclear spectroscopy. The lower energy photons that originate from electronic transitions (such as x-rays) are not suitable for this purpose mainly due to their lower penetration power as compared to γ-rays.

The standard system used for γ-ray spectroscopy consists of a NaI(Tl) scintillator and a photomultiplier tube. The scintillator converts the γ-ray photons into visible light photons, which are detected by the photomultiplier tube. A photomultiplier tube is not absolutely necessary since, in principle, any photodetector with appropriate efficiency and sensitivity can be used.

A typical γ-ray spectroscopy setup using NaI scintillator is shown in Fig.2. The shield shown in the figure has two purposes. One is to protect the personnel from radiation exposure and the other is to minimize the background in the detector. The source and the detector are positioned such that most of the γ-ray photons from thesource reach the scintillator, where they get converted into light photons. To guide the light photons towards the photomultiplier tube the scintillator is surrounded by a light reflector assembly. The light photons are then absorbed by the photocathode of the PMT, which emits photoelectrons. These photoelectrons get multiplied through the dynode assembly and ultimately lead to a measurable pulse.

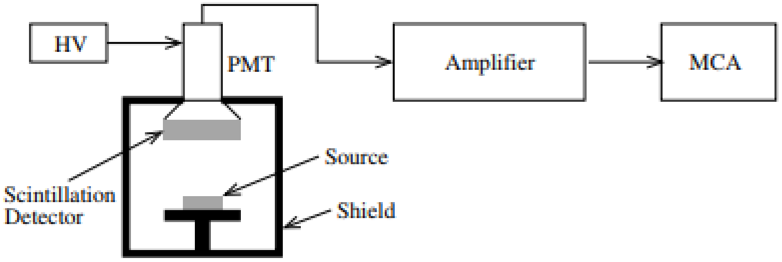

Fig.2 Block diagram of a typical γ-ray spectroscopy setup with a scintillation detector (such as NaI(Tl)) and a PMT

In the above description we have not considered the γ-ray photons that hit the shielding material. Since these photons can produce sizable effect on the output, they must be taken into account. Fig.3 shows the interaction mechanisms in the detector as well as in the surrounding shield. The γ-ray photons, as well as secondary photons, interact with the detector material and the shield through all of these modes, provided they carry enough energy.

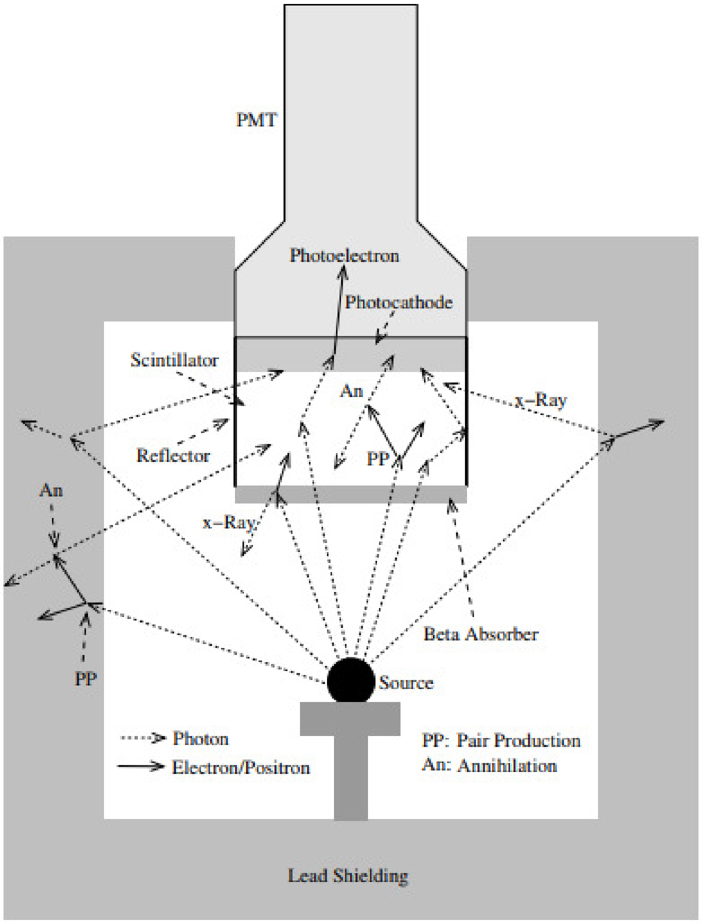

Fig.3 Photon interaction mechanisms in a γ-ray spectroscopy setup.

The usual method adopted in spectroscopy is the pulse height analysis. This technique is based on the fact that the height of the output pulse is proportional to the energy deposited in detector's active medium. Therefore if one plots the number of pulses obtained with respect to height of the pulses, it would correspond to the energy spectrum of the deposited energy. Now, since the deposited energy is directly related to the energy carried by the incident radiation, the spectrum obtained actually corresponds to that of the spectrum of the incident radiation. Pulse height analysis can be done using a single channel analyzer, though the process in that case is fairly tedious and labor intensive. The best and the most commonly used technique is the use of a multi channel analyzer or MCA. An MCA records the number of pulses in each pulse bin. The size of the pulse bin can normally be defined by the user and is usually selected according to the particular constraints of the experiment, such as required resolution and available time. The MCA spans its full dynamic range in equal width pulse bins and generally displays the output on a screen. The data can be saved, printed, or transferred. Modern MCAs can also be directly interfaced to a computer for further analysis, display, and storage of the data.

As shown in Fig.3, the photons from a radioactive source are emitted in all directions. The ones moving directly to the scintillator deposit the most energy and produce highest energy peak. The FWHM of this peak determines the resolution of the system at that particular energy. In a typical γ-ray source spectrum, other peaks are also visible. In Compton scattering, we saw that the two other peaks generally found are Compton edge and the backscatter peak. The Compton edge appears at approximately Eγ – 255 keV and corresponds to the Compton scattering of electrons at an angle such that the electrons carry away the maximum possible energy. This scattering can actually be visualized as a head-on collision of an incoming photon with an electron at rest such that the electron starts moving forward and the photon scatters back at 180^0.

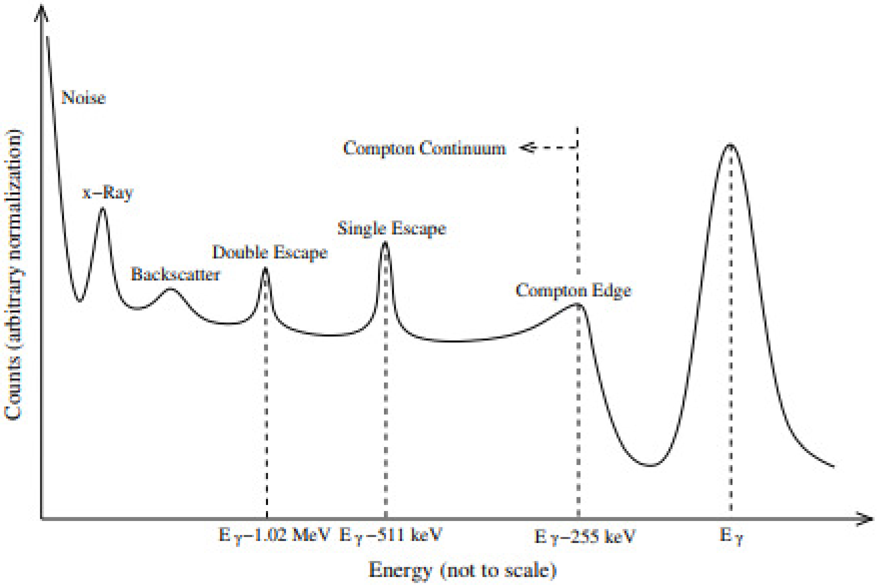

Fig.4 Typical γ-ray pulse height spectrum for a shielded setup of source and detector.

The highest energy peak and the Compton edge are sketched in Fig.4, which represents a typical energy spectrum of a radioactive source. The other notable features of the spectrum are:

- **Compton Continuum:** The curve below the Compton edge is called Compton continuum. This corresponds to the distribution of energy between incident photons and the scattered electrons during Compton scattering at different angles. Since Compton scattering is possible at all angles therefore this continuum can extend up to the beginning of the spectrum.
- **Noise Peak**: Most MCAs allow the user to gate off the noise counts from the spectrum. This is done be selecting a threshold pulse height below which no events are recorded and displayed. However in some situations one might be interested in keeping this information in the data stream. The noise peak shown in Fig.12.1.3 refers to this case.
- **Escape Peaks**: If the energy of the incident photon is greater than the threshold for pair production, that is E_{gamma}> 1.02 MeV , it can produce an electron-positron pair

in the detector as well as in the shield. The positron thus produced has very short half life and quickly combines with a nearby electron to produce photons, a process known as annihilation. Most of the annihilation events produce two photons traveling in opposite directions. Now if one or both of these photons escape the detector volume without depositing any appreciable energy. If one photon escapes and the other deposits most of its energy, it leads to a single escape peak in the spectrum (see Fig.12.1.3). The single escape peak can be easily identified since it appears at an energy of $E\gamma - 0.511$ MeV . Just like single escape peak, a double escape peak is also sometimes visible in the spectrum. This corresponds to the escape of both annihilation photons from the detector. The double escape peak appears at $E\gamma - 0.511$ MeV . It could happen that none of the annihilation photons escapes the detector. In this case the full energy carried by the incident photon gets deposited in the detector. The height of the output pulse then corresponds to the full energy peak of the spectrum ($E\gamma$ in Fig.3).

- **Shield Scattering Peaks:** These are the peaks that appear when photons hit the shielding material and are scattered into to the detector. Since these photons lose most of their energy during the collision, these peaks appear at the lower end of the spectrum. A very commonly seen shield scattering peak is the so called backscatter peak. This peak corresponds to the deposition of energy by photons that are backscattered from the shield into the detector. The backscatter peak is generally fairly broad due to scattering of photons at different angles that are close to 180^0.
- **X-Ray Peak**: When γ-ray photons interact with the high Z material of the shield, they can produce x-ray photons. The energy of these photons is substantially lower than that of the original γ-ray photons. Therefore, as shown in Fig.3, the energy deposited by them in the detector produces a peak at the lower end of the energy spectrum

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Interaction between energetic particles and matter

a) Heavy Charged Particles

A heavy charged particle (like a proton,α particle or fission fragment) has afairly definite range in a gas ,liquid or solid.The particle loses energy primarily by the excitation and ionization of atoms in its path.The energy loss occurs in a large number of small increments.The primary particle has such a large momentum that its direction is usually not seriously changed during the slowing process.Eventually it loses all its energy and comes to rest.The distance traversed is called range of the particle.

The energy loss per unit length *(-dE/dx)* is called the stopping power.The rate *-dE/dx* at which a heavy particle of chargr *ze* and speed v loses energy in an absorber of atomic number Z which contains N atoms per unit volume whose average ionization energy is *I* is given by

$$-dE/dx = \frac{z^2 e^4 N Z}{4\pi\varepsilon_0^{\,2} m_0 v^2}\left[\mathrm{In}\left(\frac{2m_0 v^2}{I}\right)-\mathrm{In}\left(1-\frac{v^2}{c^2}\right)-\frac{v^2}{c^2}\right]$$

Where m_o is the rest mass of electron.

i)Stopping power S(E). This is defined as the amount of energy lost per unit length of the medium by incident particle in the given medium. Thus,

$$S(E)= -dE/dx = N_{ion}\bar{I}$$

where N_{ion} = number of ion pairs produced per unit length.

$\bar{I}$ =Average ionization energy.

ii)Range. The range can be calculated by integrating the equation over the energies of the particle

$$R=\int_T^0 \left(-\frac{dE}{dX}\right)^{-1}.\,dE$$

b)Electrons.Electrons interact through coulomb scattering from atomic electrons,just like heavy charged particles.There are however, a number of important differences:

1.Electron travel at relativistic speeds.

2.electrons will suffer large deflections in collisions with other electrons, and therefore will followerratic paths.The range will therefore be very different from the length of the path that the electron follows.

3.very energetic electrons (E>1MeV) lose an appreciable fraction of their energies by producing continuous X-rays (also called Bremsstrahlung).The cross section for this process increases with increasing E.

C.The absorption of γ-Rays.The interaction of γ rays with matter is markedly different from that of charged particles such as α or β particles. γ-Rays are extremely penetrating so that they are able to pass through considerable thickness of matter. γ-Raysshow the exponenetial absorption in matter.if radiation of intensity I is incident upon an absorbing layer of thickness dx,the amount of radiation absorbed dI is proportional to both to dx and I.hence,

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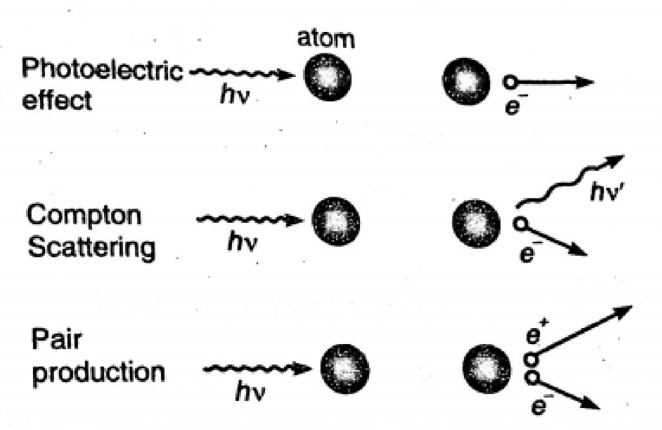
$dI=-\mu Idx$ or $I=I_oe^{-\mu x}$,where μ is a constant of proportionality which is a characteristic property of the medium,known as linear absorption coefficient.The mass absorption coefficient μ_m may be obtained by dividing μ by the density of the medium. $\mu_m= \mu_m/\rho$.The above relation gives the intensity of the beam of initial intensity I_o,after traversing a thickness of x of the homogeneous material.At low energies (0.1MeV to 25 Mev) there are three important processes through which γ photons are absorbed by matter.



i)**Photo electric effect.**

this process,the γ rays knock out electrons from inside the atoms of the absorbing material.This results in the ionization of the atoms and the emission of fluorescent radiations.einstein's equation for those photo electrons will be

$1/2mv_k^2= h\upsilon-W_k$, $1/2mv_L^2= h\upsilon-W_L$,etc

Where $h\upsilon$ is the photon energy, $v_{k,}v_L$ represents the velocities of the photoelectrons arising in the K,L shells, and $W_{k,}$ $W_{L,}$ are the binding energies of K,L shells.

ii) Pair Production.

In this process,the photon disappears and is converted to an electron –positron pair.this process can take place only when the photon energy exceeds $2m_oc^2$.The pair production process can not occur in free space and usually takes place in the presenc of a nuclear field.the nucleus recoils in this process conserving momentum.But the K.E,carried away by the nucleus is negligibly small due to its large mass compared with that of the electron.Photon energy,if any ,in excess of $2m_oc^2$ is shared as K.E by the product particles.

iii)Compton effect.

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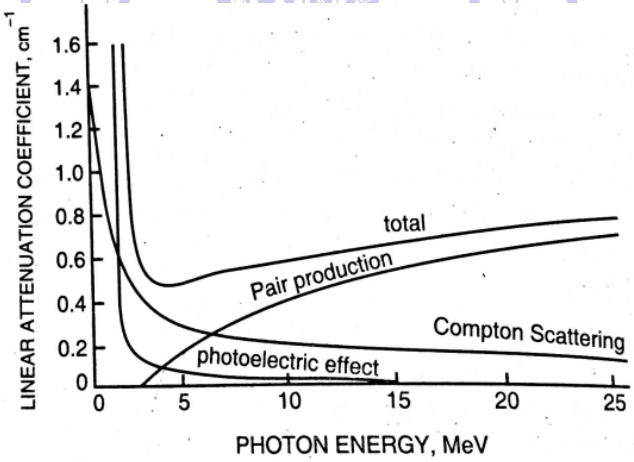
It is an elastic scattering process in which the photon imparts energy to an electron.When a photon of energy hυ strikes the perfectly free electron (at rest) is scattered ,the photon with diminished energy hυ’ is scattered at an angle θ with the direction of incident photon and the electrons recoils at an angle ϕ.The energy absorbed by these Compton electrons is only a small fraction of the total energy of the incident γ rays,unlike in the case of photoelectrons.

At loe photon energies,the photoelectric effect is the chief mechanism of energy loss.The importance of the photoelectric effect decreases with increasing energy.In the lighter elements.compton scattering becomes dominant at photon energies of a few tens of keV,whereas in the heavier ones this does not happen until photon energies of nearly 1Mev are reached.Pair production becomes increasingly likely the more the photon energy exceeds the threshold of 1.02Mev.The fig is a graph of the linear attenuation coefficient for photons in lead as afunction of photon energy.The contributions to μ of the photoelectric effect,Compton scattering and pair production are shown.



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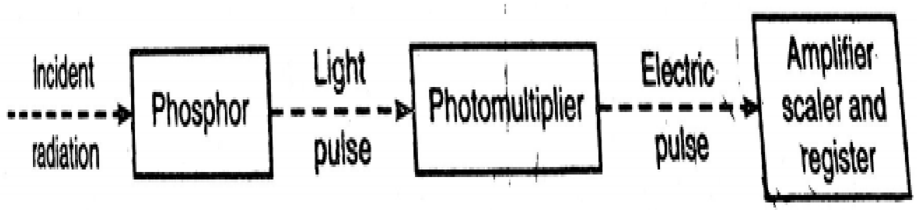
SCINTILLATION COUNTER

Principle. When high energy radiation falls on a floorescent material,it produces flashes of light called scintillations.The energy of the particle is converted to light.

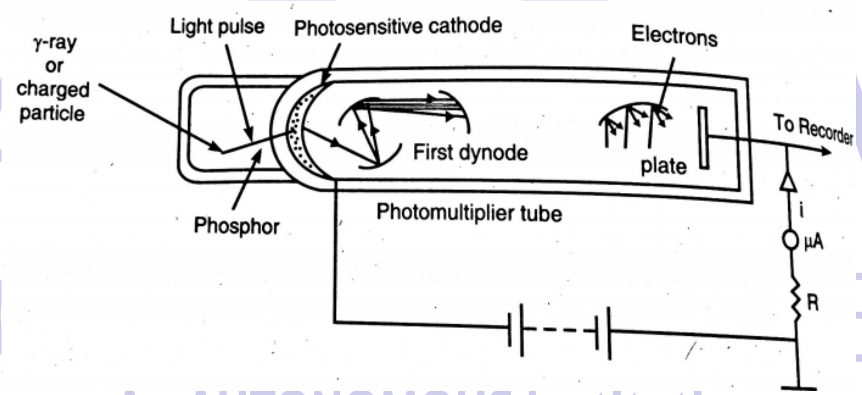


The atoms of the phosphor are excited or ionized by the energy loss of an impinging α or β or γ rays.when the atoms return to their ground states,photons are emitted,in the blue and ultraviolet regions of the optical spectrum.

The phosphor is optically coupled to the envelope of a photo multiplier tube.The output pulse from the photomultiplier is fed to a pulse amplifier folloed by a scalercircuit.

Constructions.

Fig shows the diagram of a scintillation counter.



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Working:

i.The incident radiation is absorbed by the phosphor or scintillator.The atoms or molecules are excited.

ii.when the atoms or molecules return to their ground states,Photons are emitted.

iii.the phosphor is optically coupled to the envelope of a photomultiplier tube.the photons strike the photocathode,causing the ejection of photoelectrons.

iv.As these photo electrons leave the photocathode,they are directed by a focusing electrode to the first multiplier electrode or dynode.This electrode has the property of emitting three,four or five electrons for every single electron which strikes its surface.There may be from 10 to 14 sauch multiplier stage in a given tube.hence from the emission of one single electron from the cathode, a burst of one million electrons may impinge on the final stage in the tube.(the anode0

If m is the electron multiplication factor of a single dynode stage and there are n stages,then total amplification is m^n.

v.The output pulse from the photomultiplier is fed to a pulse amplifier followed by a scaler circuit.

The following scintillators are generally used for the detection of particles and photons.

Sodium Iodide (Thallium activated).this is the most commonly used phosphor and is largely used in the study of γ rays.

Sodium Iodide crystals are usually doped with a small amount of thallium and denoted NaI(Tl).The Tl atoms act as luminescence centres. NaI(Tl)crystals are dense enough to have good efficiency of γ ray detection.

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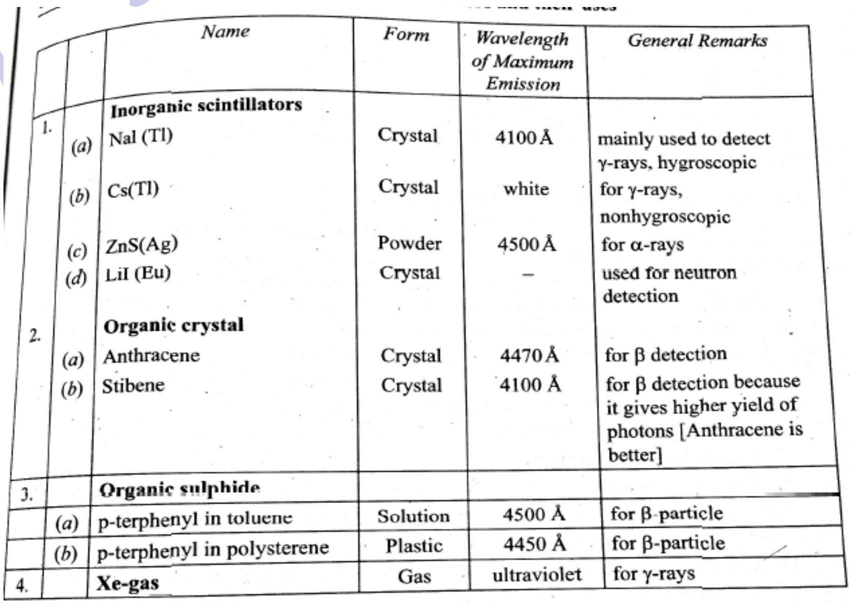
Table 18.2 common scintillators and their uses



		Name	Form	Wavelength of Maximum Emission	General Remarks
1.		**Inorganic scintillators**			
	(a)	Nal (Tl)	Crystal	4100 Å	mainly used to detect γ-rays, hygroscopic
	(b)	Cs(Tl)	Crystal	white	for γ-rays, nonhygroscopic
	(c)	ZnS(Ag)	Powder	4500 Å	for α-rays
	(d)	LiI (Eu)	Crystal	–	used for neutron detection
2.		**Organic crystal**			
	(a)	Anthracene	Crystal	4470 Å	for β detection
	(b)	Stibene	Crystal	4100 Å	for β detection because it gives higher yield of photons [Anthracene is better]
3.		**Organic sulphide**			
	(a)	p-terphenyl in toluene	Solution	4500 Å	for β-particle
	(b)	p-terphenyl in polysterene	Plastic	4450 Å	for β-particle
4.		**Xe-gas**	Gas	ultraviolet	for γ-rays

Advantages

1.The height of the pulse is proportional to the energy of the incident γ radiations.

2.In a solid scintillator,the number of atoms per unit volume is more.therefore a stronger pulse is observed in comparison to a gas scintillator..

3.The dead time of the counter is very small and hence a high counting rate is possible.

4.the pulse duration is extremely short and so the resolving power is high.

5.If a large scintillator is used,the scattered γ rays are also taken into account thereby givingrise to an increased photo peak efficiency.

Disadvantages.

1.The light output decreases with increase in energy of radiation.It also varies with the type of radiation.

The overall efficiency is low because of the complex nature of the phenomena-namely the conversion of radiation into light and then re-conversion into electrons.

Gamma well counter

Principle

A gamma counter is a machine to measure gamma radiation emitted by a radionuclide. Unlike survey meters, gamma counters are designed to measure small samples of radioactive material, typically with automated measurement and movement of multiple samples**.**

Operation

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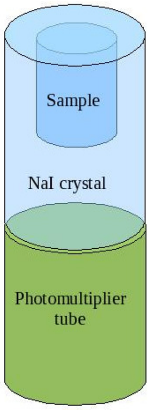
Gamma counters are usually scintillation counters. In a typical system, a number of samples are placed in sealed vials or test tubes, and moved along a track. One at a time, they move down inside a shielded detector, set to measure specific energy windows characteristic of the particular isotope. Within this shielded detector there is a scintillation crystal that surrounds the radioactive samp Gamma counters are usually scintillation counters. In a typical system, a number of samples are placed in sealed vials or test tubes, and moved along a track. One at a time, they move down inside a shielded detector, set to measure specific energy windows characteristic of the particular isotope. Within this shielded detector there is a scintillation crystal that surrounds the radioactive sample. Gamma rays emitted from the radioactive sample interact with the crystal, are absorbed, and light is emitted. A detector, such as a photomultiplier tube converts the visible light to an electrical signal. Depending on the half-life and concentration of the sample, measurement times may vary from 0.02 minutes to several hours.



If the photon has too low of an energy level it will be absorbed into the scintillation crystal and never be detected. If the photon has too high of an energy level the photons may just pass right through the crystal without any interaction. Thus the thickness of the crystal is very important when sampling radioactive materials using the Gamma Counter. Gamma rays emitted from the radioactive sample interact with the crystal, are absorbed, and light is emitted. A detector, such as a photomultiplier tube converts the visible light to an electrical signal. Depending on the half-life and concentration of the sample, measurement times may vary from 0.02 minutes to several hours.

If the photon has too low of an energy level it will be absorbed into the scintillation crystal and never be detected. If the photon has too high of an energy level the photons may just pass right through the crystal without any interaction. Thus the thickness of the crystal is very important when sampling radioactive materials using the Gamma Counter.

Applications

Gamma counters are standard tools used in the research and development of new radioactive compounds used for diagnosing and treating disease, (as in PET scanning). Gamma counters are used in radiobinding assays, radio immune assays (RIA) and Nuclear Medicine measurements such as GFR and hematocrit.Some gamma counters can be used for gamma spectroscopy to identify radioactive materials based on their output energy spectrum, e.g. as a wipe test counter.

Review Questions

Part – A

1. What are the isotopes of nucleus?
2. What is meant by Szilard Chalmer’s effect ?
3. Mention some of the radioactive tracers.
4. What are the characteristic uses of Radioactive tracers?
5. List some of the nuclear medicines and its uses.
6. Mention some of the reactions for charged particle interaction with matter.
7. Define specific ionization and linear energy transfer.
8. Define Range and Bremsstrahlung
9. What is annihilation?
10. List out the applications of Gamma ray spectrometry.
11. Give the features of Gamma ray spectrometry.

Part –B

1. Give a brief account on Szilard Chalmer’s reaction.
2. Write in brief about the use of radioactive tracers.
3. Classify the reactions for charged particle interaction with matter.
4. Give a detailed account on Nuclear medicines.
5. Describe the instrumentation and working of Gamma ray spectrometry.
6. Explain the concept of Bremsstrahlung.

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